

Dynamics of Mental Activity

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Abstract: Motivated by neuronal modeling, the mathematical foundations of consciousness developed in Miranker, Zuckerman, 2008 (M-Z) were characterized by an axiomatic theory for consciousness operators that acted on the collection of all sets. Consciousness itself was modeled as emanating from the action of such operators on the labeled decoration of a graph, the latter set theoretic construct given the characterization of experience. Since mental activity (conscious and unconscious) is a time dependent process, we develop a discrete time dependent version of the theory. Specification of the relevant dynamics illuminates and expands the development of the mathematical framework in M-Z upon which our study of consciousness rests.

We review the Aczel theory for decorating labeled graphs, in particular that theory's application to the M-Z foundations. The relevant neuronal modeling concepts and terminology are also reviewed. A number of simple examples are presented. Then an extension of our considerations from graphs to multigraphs is made, since the latter represent a more accurate model of neuronal circuit connectivity. The dynamics are crafted for non-well-founded constructs by development of a hierarchy of systems, starting with the McCulloch-Pitts neuronal voltage input-output relations and building to a dynamics for the cognitive notions of memes and themata; these latter corresponding to aspects of decorations of labeled graphs associated with neural networks. We conclude with a summary and discussion of the semantics of the cognitive features of our development: memes, themata, quale, consciousness operators, awareness field...

Key words: anti foundation axiom, consciousness, M-Z Theory, mental dynamics, neural networks, non-well-founded sets

1. Introduction

The mathematical foundations of consciousness developed in Miranker, Zuckerman, 2008 (hereafter referred to as M-Z) motivated by neuronal modeling are characterized by an axiomatic theory for a collection of so-called consciousness operators that act on the collection of all sets. Consciousness itself was modeled as emanating from the action of such an operator on the labeled decoration of a graph, the latter set theoretic construct given the characterization of experience. The interplay of the Platonic perspective on the real (the physical) and the ideal (usually called the Platonic), which formed the underpinning of the approach (Plato, 360 BCE), was modeled by the quality of sets being well-founded or not-well-founded. We use this Platonic dichotomy to characterize those aspects of our modeling that correspond to observable, measurable features of nature (such as voltages, synaptic weights...) as the real (physical), while those aspects of our modeling that correspond to virtual features (such as perceptual experience, qualia...) are characterized as the ideal Platonic, or simply the Platonic. This Platonic distinction has been a basic constituent of much of the philosophical discourse on consciousness with the Cartesian notions of the *res extensa* and the *res cogitans* being a conspicuous example (Descartes, 1637). The present work along with M-Z contain developments that take the Platonic dichotomy to the level of mathematical modeling and analysis.

The ability to study the interior of a set from the outside plays a key role in the development. It was the consciousness operators (see the Appendix) introduced in M-Z by means of which this ability was implemented. These ideas are characterized in Fig. 1.1 (Fig. 5.2 in M-Z) that shows the flow of information from sensory input in neuronal networks to conscious experience. The upper boxes in the figure describe the syntactic level, the lower the semantic level of the modeling. Shading in the figure distinguishes the ideal Platonic realm from the physical.

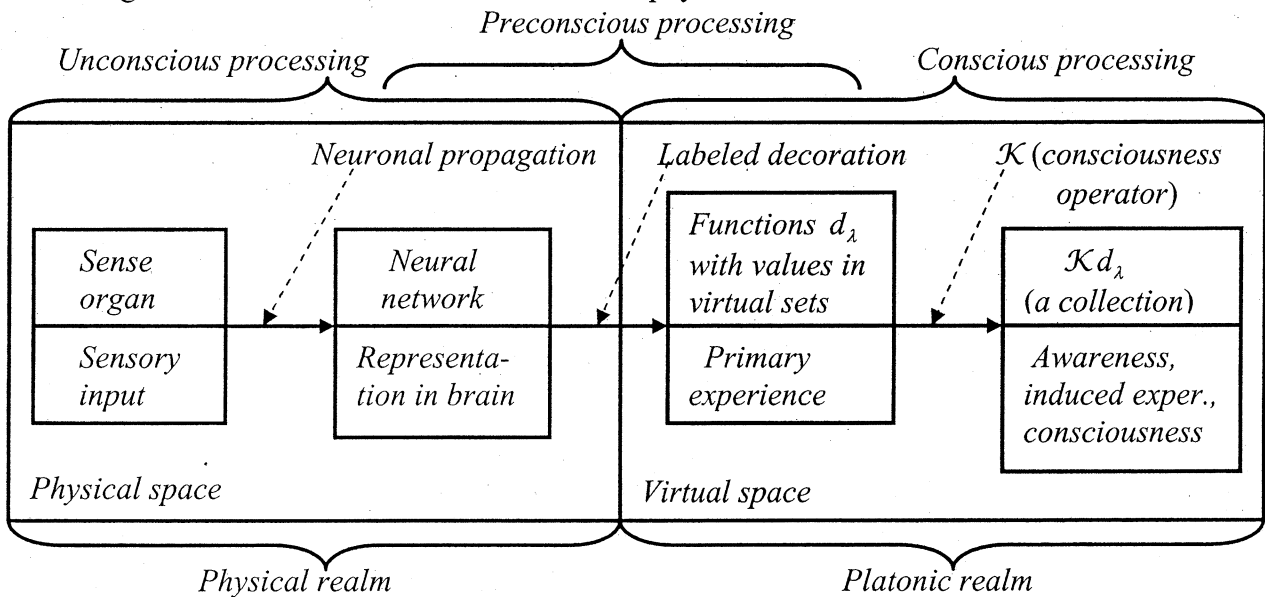


Figure 1.1: Consciousness: Syntactic and semantic views of the processing proceeding from the physical realm to the Platonic realm.

A non-well-founded graph is taken to model neural network connectivity in the brain. The corresponding labeled decorations of such graphs are not recursively computable. They are schematized in the box “*Functions d_λ with values in virtual sets*” in Fig. 1.1. Connectivity is a static feature, and since consciousness is a time dependent process, it is essential to develop a time dependent version of these decorations; an aspect of the theory left open in M-Z. Moreover specification of the relevant dynamics (by dynamics, we shall always mean discrete dynamics) will illuminate and expand the development of the mathematical framework in M-Z upon which the modeling of mental activity (conscious and unconscious) rests. In particular, the dynamics reveal a quality of reversibility among many aspects of the framework. Specification and study of these dynamics is a principle objective of this work, which itself is composed of two parts.

Part I, consisting of Sects. 2-6, contains a static version of our theory, while Part II, consisting of Sects. 7-12, deals with the dynamics. In Part I we review the Aczel theory, a methodology for decorating labeled graphs. Since the extension of this theory in M-Z is motivated by questions of neural circuitry and consciousness studies, some of the relevant neuronal modeling concepts and terminology are also reviewed. A number of simple examples are presented. Then an extension of our considerations from graphs to multigraphs is made, since the latter represent a more accurate model of neuronal circuit connectivity. In Part II the M-Z Theory is exploited and extended from its static description in M-Z to the dynamic version needed for a deeper understanding of the structure of mental activity and in particular, conscious experience. These dynamics are crafted for non-well-founded constructs by development of a hierarchy of systems, starting with the McCulloch-Pitts neuronal voltage input-output relations (Haykin, 2008) and building to a dynamics for the cognitive notions of memes and themata; these latter corresponding to aspects of decorations of labeled graphs.

Part I: Statics

We begin in Sect. 2.1 with a review of the notions and terminology of sets and graphs that are used. Our considerations involve non-well-founded (NWF) sets, the axiom of anti-foundation and some key operators. We continue in Sect. 2.2, collecting the notions and terminology of neural networks that motivate and in turn are modeled by the development. A description of the M-Z Theory for developing histograms and decorations that emerge from a neural network is included.

In Sect. 3, a number of simple examples to illustrate the M-Z Theory is given. Also illustrated is the Russell operator, a key example of a consciousness operator.

In Sect. 4, the notions of memes and themata are introduced, and their connection to our graphical constructs is specified. Examples of these cognitive and graphical constructs are given. These include examples of the neuronal data histograms introduced in M-Z, the latter framing the technical foundation of our approach.

A pair of neurons may exchange information through multiple connections. So in Sect. 5, we extend the development from graphs to multigraphs and introduce a number

of simple examples. This is followed by a development of so-called special graphs relative to which memes and themata are reciprocally specifiable.

Beginning in Sect. 6, multigraphs are restricted so that the decoration of any node is NWF, enabling thereby the development of reversibility. This allows for the retrieval of the M-Z histogram from the M-Z decoration. Examples are given. A particular consciousness operator is used to develop an example of self-awareness of a neural state.

Part II: Dynamics

In Sect. 7, we begin with the description of the dynamics of neuronal input-output relations in our framework. We develop a dynamics of histograms, which supervene upon the underlying McCulloch-Pitts input-output voltage dynamics of a model neuron. A notion of nodal activity along with its dynamics is introduced. A reciprocal relationship among the voltage, histogram and activity dynamics is established.

In Sect. 8, the auxiliary notions of a crop and of crop space are introduced in order to conceptualize the set of all M-Z histograms. These constructs along with the related crop dynamics abstract and simplify the development. A further abstraction called the mean (of a crop) and its dynamics is introduced. The reciprocal relationship of mean and crop dynamics is established.

In Sect. 9, we introduce the Aczel transform, a construct that informs the development of decoration dynamics. Then a review of the dynamics thus far developed, including an illustrative diagram, leads to the development of the reciprocal relationships among voltage, crop and decoration dynamics.

In Sect. 10, we introduce the space of all the memes (M-Z decorations) associated with a given weighted multigraph. We also introduce the space of the associated themata. This enables us to interrelate all eight of the spaces of our development.

In Sect. 11, we introduce a well-defined dynamics for memes. The relationship of these dynamics to those already introduced is displayed in a three dimensional commutative diagram. Following this the associated dynamics for themata (although not single valued) is specified.

The foundations of consciousness developed in M-Z emanate from a framework of static mappings associated with the M-Z Theory. These mappings describe an irreversible passage from the physical realm to the Platonic. Dynamics enable the introduction of reversibility in this passage along with a more detailed description of mental activity and the relationship between those realms. In Sect. 12, we start with a critical summary of those dynamical features developed in Sects. 7-11 that reside in the physical realm, and we conclude with a discussion of the semantics associated with the Platonic aspects of those features.

PART I: STATICS

2. Preliminaries

We shall be concerned with sets and graphical constructs associated with sets, and so in Sect.2.1, we introduce some relevant ideas and nomenclature. We refer to Aczel, 1988 (hereafter referred to as Aczel) and to M-Z for additional details. In Sect 2.2, we give a description of the M-Z Theory, beginning by introducing the supporting concepts and terminology of neural networks.

2.1 Sets, graphs and consciousness operators

A set S is a primitive that obeys the axioms of set theory. In particular

$$S \in \mathcal{S} = \mathcal{S}_{wf} \cup \mathcal{S}_{nwf}, \quad (2.1)$$

where \mathcal{S} is the class of sets, \mathcal{S}_{wf} is the class of well-founded (WF) sets, and \mathcal{S}_{nwf} is the class of non-well-founded (NWF) sets. (WF sets and NWF sets are specified in Def. 2.2.)

A *graph*¹ will consist of a collection N of *nodes* and a collection E of *edges*, each edge being an ordered pair (n, n') of distinct nodes. We have no knowledge of the nature of the elements of N . If (n, n') is an edge, we shall write $n \rightarrow n'$ and say that n' is a *child* of its *parent* n . A *path* is a sequence (finite or infinite)

$$n_0 \rightarrow n_1 \rightarrow n_2 \rightarrow \dots \quad (2.2)$$

of nodes $n_0, n_1, n_2 \dots$ linked by edges $(n_0, n_1), (n_1, n_2) \dots$

A *pointed graph* is a graph in which a node called its *point* has been distinguished. A pointed graph is *accessible* (is an accessible pointed graph (APG)) if for every node n different from the point, say n_0 , there is a path $n_0 \rightarrow n_1 \rightarrow \dots \rightarrow n$ from n_0 to the node n .

A *decoration* of a graph is an assignment of a set to each node of the graph so that the elements of the set assigned to a node are the sets assigned to the children of that node. Equivalently, a decoration is a set valued function $d: N \rightarrow \mathcal{S}$, where

$$\forall R \in N, \quad dR = \{dQ \mid R \rightarrow Q\}. \quad (2.3)$$

The set dR is called the decoration of the node R .

A *picture* of a set A is an APG that has a decoration in which A is assigned to the point. A set may have many different pictures. A *labeling* of an APG is a mapping

¹ We shall extend our considerations from graphs to multigraphs beginning in Sect. 5.

$\lambda: N \rightarrow S_{wf}$, that is, an assignment of a WF set to each node. A labeled APG to be denoted by the pair (Γ, λ) will be called an LAPG. A labeled decoration of an LAPG is a set valued function $d_\lambda: N \rightarrow S$, where

$$\text{Aczel equation:} \quad d_\lambda R = \{d_\lambda Q \mid R \rightarrow Q\} \cup \lambda R, \quad \forall R \in N. \quad (2.4)$$

A decorated LAPG is called a DLAPG. The map $\delta: \text{DLAPG} \rightarrow S$ takes the DLAPG into the decoration $d_\lambda P$ of the point P of the APG. We sometimes refer to $d_\lambda P$ as dP .

Remark 2.1: Throughout we make critical use of the *anti-foundation axiom* (AFA) of Aczel, 1988. This axiom, *every graph has a unique decoration*, is a contemporary replacement of the axiom of foundation (AF) of Von Neumann, 1925. It is a consequence of the AFA that every LAPG has a unique decoration.

The Russell operator \mathcal{R} is given by $\mathcal{R}x = \{y \in x \mid y \notin y\}$. It is a key example of a consciousness operator, the notion of which is elaborated upon in both Sect.12 and the Appendix.

We now formalize the notions of a WF set, and then we introduce the associated operator \mathcal{W} that selects the WF part of a set.

Definition 2.2 (WF set²): A set T is WF if $\forall X \subseteq T, X \neq \emptyset, \exists a \in X$ such that $a \cap X = \emptyset$. T is NWF otherwise. (See Hrbacek, Jech, 1999 and Jech, 2002.)

Alternative: If T is WF, then $\forall x \in T$, there does not exist an infinite chain of sets where $x \ni y_1 \ni y_2 \ni \dots \ni y_n \ni \dots$

Note that Def. 2.2 precludes the existence of loops in the picture of a WF set.

Definition 2.3 (WF part-of-a-set operator): The operator \mathcal{W} that “takes the WF part of a set” is given by:

$$\mathcal{W}x = \{y \in x \mid y \in S_{wf}\}. \quad (2.5)$$

Properties of the operator \mathcal{W} and of the set $\mathcal{W}x$ are the subjects of the following lemma.

Lemma 2.4: 1. $\forall x \in S, \mathcal{W}x$ is WF and 2. $\mathcal{W}x \subseteq \mathcal{R}x \subseteq x$, where \mathcal{R} is the Russell operator.

Proof: 1. An immediate consequence of Def 2.2, alternative.

2. Since $y \in S_{wf} \Rightarrow y \notin y$, then $\mathcal{W}x \subseteq \mathcal{R}x \subseteq x$

² For completeness we state the foundation axiom of Von Neumann: *Every set is WF*. Recall that we have replaced AF by AFA. (See Remark 2.1.)

2.2 Neuronal modeling, M-Z theory

We now introduce some notions relating the set and graphical constructs of Sect. 2.1 to neuronal modeling. In Table 2.2 we collect terms that our subsequent discussion will specify and make use of. Then a description of the M-Z Theory follows.

Construct	Syntax	Semantics	Comment
APG	(Γ, P)	<i>Neural Net</i>	Neuronal assembly
NAPG	(Γ, v, w, P)	<i>Neural State</i>	Assembly of Mc-P neurons at an instant of time
LAPG	$(\Gamma, \lambda_{v,w})$	<i>Neural Labeling</i>	$\forall Q \in N, Q \mapsto \lambda_{v,w} Q = H_{w _{Q,v}}$ $H_{w _{Q,v}}$ is a histogram
DLAPG	$(\Gamma, d_{\lambda_{v,w}})$	<i>Neural Decoration</i>	$\forall Q \in N, Q \mapsto d_{\lambda_{v,w}}$ Meme: An instantiation of a concept
Set	$d_{\lambda} P \in \mathcal{S}$	<i>The set decorating the point</i>	Thema: A thought-meme, a state of mind, the set whose picture is the meme

Table 2.2: Terms used in the development

A *neural net*, denoted (Γ, P) is a particular type of graph built upon an abstracted form of a representation of a living neuron called the McCulloch-Pitts model (Mc-P), a simple model, chosen for clarity (see (7.1) and Haykin, 2009). Each node of (Γ, P) corresponds to a (model) neuron and each directed edge corresponds to an *efferent (output) synapse* of that neuron (parent node). For convenience we shall at first restrict our attention to neural nets whose graphs are APGs. This restriction will be relaxed later.

M-Z Theory: Let us now consider a finite APG along with a collection of so-called voltage v and weight w data, that data being

$$\text{Voltage and weight data: } (v : N \rightarrow \{0,1\}, w : E \rightarrow \mathbf{Q}). \quad (2.6)$$

Here \mathbf{Q} denotes the set of rational numbers, a choice made for reasons of definiteness and clarity. A node with the datum $v = 1$ corresponds to a *firing or active neuron*. An edge with weight w corresponds to an efferent (output) synapse with weight w . This construct is a neural net at a fixed time instant with the efferent voltages and the synaptic weights are specified (at that time). This construct, denoted NAPG, will be called a *neural state*, and it will be represented by the symbol (Γ, v, w, P) . An NAPG models an assembly of living neurons at an instant of time. Now construct a labeling of (Γ, v, w, P) , namely construct the LAPG denoted by the pair $(\Gamma, \lambda_{v,w})$, where

$$\text{Histogram: } \lambda_{v,w} Q = H_{w|_{Q,v}}, \quad \forall Q \in N. \quad (2.7)$$

Here the set $H_{w|_{Q,v}}$ is the histogram of $w|_{Q,v}$, namely the histogram (a WF set) of the weight data of an active node and the edges that emanate from such a node. (See M-Z for a discussion of histograms.) $E_{P,v}$ will denote the set of so-called active edges; those edges emanating from active nodes that lead into P :

Active edges:
$$E_{P,v} = \{(Q,P) | v(Q)=1\}. \quad (2.7)$$

Note that $H_{w|_{Q,v}} \subseteq (\text{ran } w) \times |N|$ (see M-Z). Finally decorate the LAPG $(\Gamma, \lambda_{v,w})$, using the set valued function $d_{\lambda_{v,w}} : N \rightarrow \mathcal{S}$. The resulting DLAPG, denoted by the pair $(\Gamma, d_{\lambda_{v,w}})$ is a *neural decoration* of the labeled neural state (the LNAPG). The mapping $d_{\lambda_{v,w}}$ is called the M-Z function. It is specified by the following M-Z equation obtained by replacing λ by $\lambda_{v,w}$ in the Aczel equation (2.4).

M-Z equation:
$$d_{\lambda_{v,w}} R = \{d_{\lambda_{v,w}} Q | R \rightarrow Q\} \cup \lambda_{v,w} R, \quad \forall R \in N. \quad (2.8)$$

Note that in passing from (Γ, v, w, P) to $(\Gamma, d_{\lambda_{v,w}})$, the labeling data is replaced by the set valued function $d_{\lambda_{v,w}}$. The construct $(\Gamma, d_{\lambda_{v,w}})$ is associated with the NAPG. Since the histogram makes no use of the data concerning inactive nodes, this passage occurs with a *loss of information*. This information loss is illustrated by example 3 in Sect. 3.

Model: The NAPG, LAPG and DLAPG are successively more elaborate constructs upon which we base our model of the state of an assembly of neurons. Each abstracts the form and activity of an assembly of Mc-P neurons at an instant of time.

Platonism: All of the constructs in Table 2.2 are mathematical abstractions, that is, they are ideal or Platonic entities. Those that model physical objects may in their manifestation as a graph... be viewed as objects in reality, but when so viewed, we are in fact contemplating the modeled object itself. Memes and themata are specified in Sect. 4. A meme may be Platonic or physical in the sense just described, but since a thema is a set that models a state of mind, it is taken as strictly Platonic. (See Fig 12.3.)

3. Specific Examples of Employing the M-Z Theory

In this section, we give a number of illustrative applications for employing the M-Z equation. Properties of the Russell operator, a key consciousness operator (see the Appendix) are also illustrated.

Applications

1. Consider the graph consisting of a single node P having no edges and with a zero voltage datum. Since P is inactive and has no children,

$$\delta P = \emptyset. \quad (3.1)$$

2. Consider the following neural net (Γ, P) with one node P and one edge (P, P) .



When supplied with data $v(P)$ and $w(P, P)$, it becomes a neural state (Γ, v, w, P) . When (Γ, v, w, P) is labeled and then decorated using the M-Z equation, it becomes a neural decoration $(\Gamma, d_{\lambda_{v,w}})$. δP is specified by that equations as follows.

$$\delta P = \{\delta P\} \cup H_{P,v}. \quad (3.2)$$

i) In particular, if $v(P)=0$,

$$\delta P = \{\delta P\} \cup \emptyset. \quad (3.3)$$

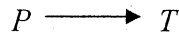
We see that decorating the neural net produces the *Quine atom* $\delta P = \{\delta P\}$. The Quine atom, denoted by Ω , is specified by the relation $\Omega = \{\Omega\}$. Note that in the case $v=0$ at hand, the DLAPG, having no information about w , does not specify the NAPG.

ii) If $v(P)=1$,

$$\begin{aligned} \delta P &= \{\delta P\} \cup \{(w(P, P), 1)\} \\ &= \{\delta P, (w(P, P), 1)\} \\ &= \mathcal{D}(w(P, P), 1), \end{aligned} \quad (3.4)$$

where \mathcal{D} is the duality operator³.

3. Consider the following APG with nodes P , the point, and T , the point's child.



i) If $v(P)=v(T)=0$, and $w(P, T)=0$, where (P, T) denotes the edge from P to T , then $\{\emptyset\}$ is the set decorating the point. Then $\{\emptyset\} \bullet \rightarrow \bullet \emptyset$ is a decoration of the graph. That is, $\{\emptyset\} \bullet \rightarrow \bullet \emptyset$ a picture of the set, and so, a picture of the Von Neumann ordinal 1.

ii) Let $v(P)=1$. Then independently of the value of $v(T)$, the set $E_{T,v} = \{(P, T)\}$, and $E_{P,v} = \emptyset$. Now consider the mapping $w_{T,v} : E_{T,v} \rightarrow \mathbf{Q}$. Let s denote the synaptic weight associated with the edge (P, T) :

$$s = w_{T,v}(P, T) \in \mathbf{Q}. \quad (3.5)$$

Since s arises exactly once,

$$H_{w_{T,v}} = \{(s, 1)\}, \quad (3.6)$$

for the histogram $H_{w_{T,v}}$ (previously denoted as $H_{w|_{T,v}}$), where $(s, 1)$ is an ordered pair. We

³ Let x^* be the unique solution to $x^* = \{x^*, x\}$. Then $x^* = \mathcal{D}x$ defines the duality operator \mathcal{D} .

use the notation (A,B) to denote either the edge from A to B or an ordered pair; the meaning clear from the context (see M-Z).

Since there is no edge leading to P , the histogram $H_{w_{P,v}}$ is empty,

$$H_{w_{P,v}} = \emptyset. \quad (3.7)$$

The M-Z equations are

$$d_\lambda P = \{d_\lambda T\} \cup \emptyset \quad (3.8)$$

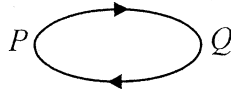
and

$$d_\lambda T = \emptyset \cup H_{w_{T,v}} = \{(s,1)\}. \quad (3.9)$$

Solving these gives

$$d_\lambda P = \{\{(s,1)\}\}. \quad (3.10)$$

4. Now consider the neural state corresponding to the APG in the following diagram.



Here (a) the set of nodes $N = \{P,Q\}$, (b) the set of edges $E = \{(P,Q),(Q,P)\}$, and (c) the data $v : N \rightarrow \{0,1\}$ and $w : E \rightarrow \mathbf{Q}$.

i) If the voltages at both P and Q are zero, the histograms are empty, and so employing both M-Z and the anti-foundation axiom (AFA, see Aczel), we see that decorating the neural net produces the Quine atom at each node.

ii) Take $v(P)=1$ and $v(Q)=0$. Then the set of active edges $E_{P,v} = \emptyset$. The histogram $H_{P,v} = \emptyset$, since the voltage at Q is zero. Also $E_{Q,v} = \{(P,Q)\}$ is the set containing the edge (P,Q) . $H_{Q,v} = \{(w(P,Q),1)\}$, since there is exactly one edge connecting P to Q .

Then the M-Z equations are

$$d_\lambda P = \{d_\lambda Q\} \cup H_{P,v} \quad (3.11)$$

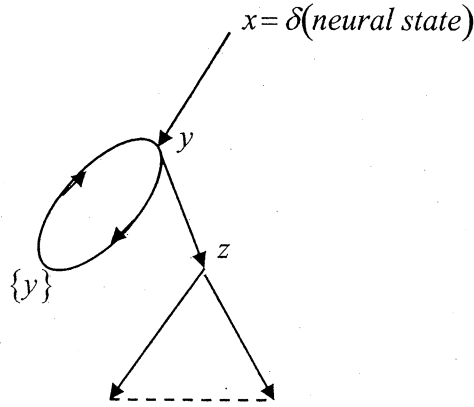
and

$$d_\lambda Q = \{d_\lambda P\} \cup \{(w(P,Q),1)\}. \quad (3.12)$$

Recall that $H_{P,v} = \emptyset$. Then solving this M-Z system gives

$$d_\lambda Q = \{\{d_\lambda Q\}, (w(P,Q),1)\}. \quad (3.13)$$

Now introduce the abbreviations $x = d_\lambda P$, $y = d_\lambda Q$, $z = (w(P,Q),1)$, in terms of which we have the following diagram generated by the M-Z equations (3.10) - (3.12), which in particular is a picture of x . Sets of the labeled decoration are indicated in the diagram.



Notation: Throughout, a triangle with a dashed base is shorthand for a picture of the set appearing at the vertex of that triangle.

iii) Take both voltages $v(P)=v(Q)=1$. Then $E_{P,v} = \{(Q,P)\}$, and $H_{P,v} = \{(w(Q,P),1)\}$. $E_{Q,v}$ and $H_{Q,v}$ are as in (ii). The M-Z system

$$d_\lambda P = \{d_\lambda Q\} \cup H_{P,v} \quad (3.11)$$

and

$$d_\lambda Q = \{d_\lambda P\} \cup H_{Q,v} \quad (3.12)$$

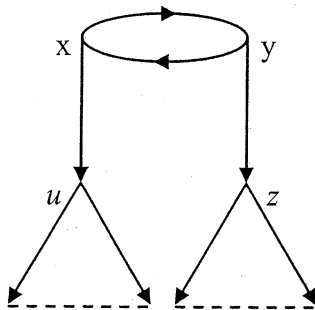
now becomes

$$d_\lambda P = \{d_\lambda Q\} \cup \{(w(Q,P),1)\} \quad (3.13)$$

and

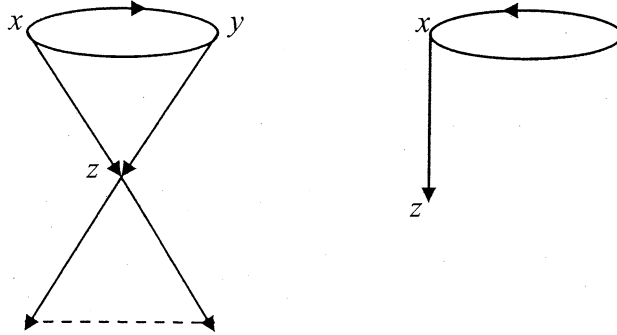
$$d_\lambda Q = \{d_\lambda P\} \cup \{(w(P,Q),1)\}. \quad (3.14)$$

Setting $x = d_\lambda P$, $y = d_\lambda Q$, $u = (w(Q,P),1)$ and making use of AFA, we assemble the, DLAPG, a picture of the set $d_\lambda P$, in the following diagram and consider two cases.



Case a) $u \neq z$: In this case $x \neq y$, since $z \in y$, but $z \notin x$. Moreover since the set x is NWF, but the set u is WF, we have $x \neq u$. So x contains u and y but not itself. This example allows us to illustrate the action of the Russell operator \mathcal{R} . In particular since x contains no element that is a member of itself, $\mathcal{R}x = x$. Likewise, $\mathcal{R}y = y$.

Case b) $u = z$: In this case, redrawing the picture (shown below on the left) and appealing to the uniqueness in the AFA, we conclude that $x = y \neq z$. The inequality follows, since z is WF. This also implies that $z \notin z$. Then $x = \{x, z\}$, a picture of x , which is shown below on the right.



These observations allow us to conclude that

$$\mathcal{R}x = \{z\} \neq x, \tag{3.15}$$

the last inequality following since $\{z\}$ is a proper subset of x .

4. Memes and Themata

A meme (Seemon, 1921, Dawkins, 1976, Blackmore, 1999) is a concept or an instantiation of a concept. A thema (Miranker 2008) is an instantiation of a meme as a conscious experience (a thought-meme). We shall associate these cognitive constructs with our development, and in doing so; they should more properly be called neural memes and neural themata, respectively. For clarity, we shall drop the adjective neural in these cases. For the context of our development, we compose novel set theoretic specifications for these constructs, beginning with the following definitions. (Semantic interpretations of memes and themata are given in Sect. 12.)

Definition 4.1 (Meme): A *meme* is a neural decoration (DLAPG), a construct, the mapping d_λ associated with a neural state (NAPG) of the LAPG into S .

Since there can be a loss of information in passing from an NAPG to a DLAPG, the latter is not determinant of the former. Example 2(i) in Sect. 3 shows a case of such a loss of information. A meme is a Platonic construct, but as we have seen, as a neuronal model, it could be viewed as correlated with a physical instantiation, namely the physical neural state modeled by the construct.

Definition 4.2 (Thema): A *thema* is the set δP that decorates the point P of the graph decorated by the meme.

A thema is strictly a Platonic construct (Miranker, 2008). Note that a thema may correspond to an arbitrary number of (related) memes. (See the examples that follow.)

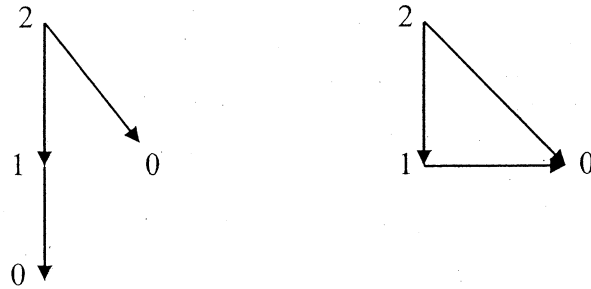
The following informal observation supplies semantic content to these concepts.

Observation 4.3: Viewing a meme as a field of sets (in the decoration), the thema as the representation of each of the memes can be viewed as the literal theme of that field.

Example 2, Sect. 3 shows that $(\Gamma, d_{\lambda, w})$ is a meme whose thema is the Quine atom. (See Sect. 3, example 2i.)

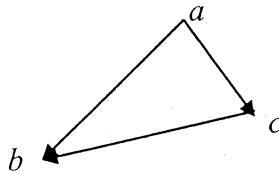
Examples

1. The following are two decorated APGs. The symbols at each node represent the sets of the decoration, each such set being the indicated Von Neumann ordinal.



These decorated APGs represent two distinct pictures of the set corresponding to the Von Neumann ordinal 2, and both have the integer 2 as a thema. If we label these graphs using zero weight and voltage data throughout, the APGs become NAPGs. Should any of the voltages be changed to unity, the decorations change, and we need to appeal to the M-Z equation for their specification, in particular to determine the new memes and themata. This example shows that *a single thema may correspond to each of a collection of multiple memetic instantiations.*

2. Consider the following APG with nodes called a , b , and c . The symbol a denotes the point of the APG.



By labeling the graph with voltage and weight data and then decorating the result, we obtain a DLAPG. One of the resulting M-Z equations is

$$da = \{db, dc\} \cup H_{a,v}, \quad (4.4)$$

where $H_{a,v}$ is the histogram of $w|_{E_{a,v}}$. Since there are no edges directed into a , $E_{a,v} = \emptyset$, and so, the full set of M-Z equations is

$$da = \{db, dc\},$$

$$db = H_{b,v}$$

and

$$dc = \{db\} \cup H_{c,v}. \quad (4.5)$$

(i) Now suppose that all three voltages equal unity. (For clarity we shall drop the second subscript v .) Let us determine H_b , the histogram of $w|_{E_b}$. $E_b = \{(a,b), (c,b)\}$, the set consisting of the two indicated edges. Then there are two relevant weights (one for each of these edges). Let us suppose that these weights are different, i.e., that $w(a,b) \neq w(c,b)$. Then

$$H_b = \{(w(a,b), 1), (w(c,b), 1)\}, \quad (4.6)$$

and

$$H_c = \{(w(a,c), 1)\}. \quad (4.7)$$

Then

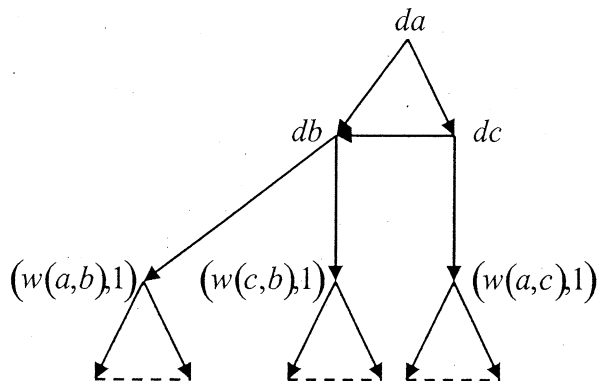
$$db = \{(w(a,b), 1), (w(c,b), 1)\}, \quad (4.8)$$

and

$$dc = \{db, (w(a,c), 1)\}. \quad (4.9)$$

The following diagram shows the DLAPG corresponding to this NAPG. It is a meme with da as its thema.

(ii) Other cases wherein not all voltages are unity can be readily derived from this case. For instance, take the case where $v(b) = 1$, but $v(a) = v(c) = 0$. Since both source voltages are zero, all three histograms are empty. Then $db = \emptyset$, $dc = \{db\} = \{\emptyset\}$, and $da = \{\emptyset, \{\emptyset\}\}$. We recognize the thema da as the Von Neumann ordinal 2, and one of the two examples described earlier as Example 1 in this section.



5. Multigraphs

A pair of living neurons may send information through multiple connections (synapses), and in order for our modeling to capture this feature; we extend our consideration, in particular, the M-Z Theory, from graphs to multigraphs. For clarity we restrict ourselves to finite multigraphs.

5.1 Definitions and examples

Definition 5.1 (Multigraph): A multigraph is a quadruple (N, A, s, f) . N is a set of nodes, and A is a set of arrows, where nodes and arrows are primitives. s and f are mappings from arrows to nodes. $s(\text{arrow})$ is a node called the initial point or the source of an arrow, and $f(\text{arrow})$ is a node called the final point or the target of an arrow. The mapping $s \times f : A \rightarrow N \times N$ is not necessarily 1-1. (Here \times denotes the Cartesian product.) By an edge in the multigraph, we mean an ordered pair of nodes connected by at least one arrow. That is, there is an arrow such that $s(\text{arrow})$ is the first of these nodes and $f(\text{arrow})$ is the second of these nodes. The set E_A of edges is defined by

$$E_A = (s \times f)(A) \subseteq N \times N. \quad (5.1)$$

So an edge corresponds to a collection of all of the arrows associated with an ordered pair of nodes. Hereafter we write $a \rightarrow b$ if $\exists \alpha \in A$ such that $s(\alpha) = a, f(\alpha) = b$.

An accessible pointed multigraph will be called an APM. An NAPM will denote the analog of an NAPG when the neural state for the latter (corresponding to voltage and weight data) is replaced by a corresponding multigraph supplied with arrow data (a weight function $w : A \rightarrow \mathbf{Q}$) and node data (a voltage function $v : N \rightarrow \{0,1\}$).

The following definition specifies the M-Z equations for decorating an LAPM (the multigraph analog of an LAPG).

Definition 5.2 (M-Z equation for an NAPM): Let a, b, \dots denote the nodes of a multigraph. Then the decoration of the LAPM is specified as follows.

$$da = \{db \mid a \rightarrow b\} \cup H_{a,v}, \quad (5.2)$$

and

$$H_{a,v} = \text{histogram of } w \Big|_{A_{a,v}} = H_{w_{a,v}}. \quad (5.3)$$

Here the active arrows, $A_{a,v}$ are specified as follows.

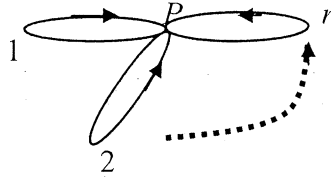
$$A_{a,v} = \{ \text{arrows } \alpha \mid f(\alpha) = a, v(s(\alpha)) = 1 \}. \quad (5.4)$$

The collection A_a of arrows terminating in a node a will play a key role in what follows.

We now give some examples both of NAPMs as well as of the sets decorating them to produce DLAPMs (the multigraph analog of DLAPG). We shall see that when specifying the histograms, it is the arrows that are directly involved with the edges playing a subordinate role.

Examples

1. Now consider the following example of an APM with one node P (the point) and r arrows, the latter arbitrarily numbered with $j = 1, \dots, r$.



For the set of arrows A_p we have.

$$A_p = \{r \text{ arrows } j \mid s(j) = P, t(j) = P\}. \quad (5.5)$$

If $v(P) = 0$, the node P is inactive, and so δP is the Quine atom.

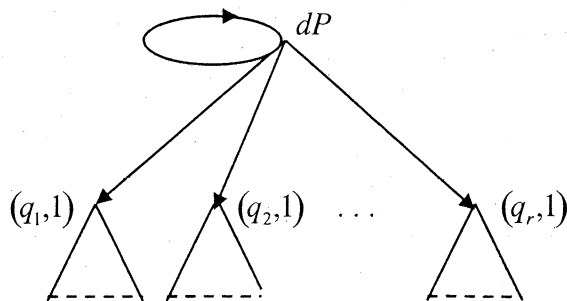
Now let $v(P) = 1$: Then the point is labeled according to the data $v(P)$ and the associated weights. Take the weights as a vector of r distinct rationals q_j , $j = 1, \dots, r$. Then for the histogram $H_{p,v}$ at P , we have

$$H_{p,v} = \{(q_1, 1), \dots, (q_r, 1)\}. \quad (5.6)$$

Then

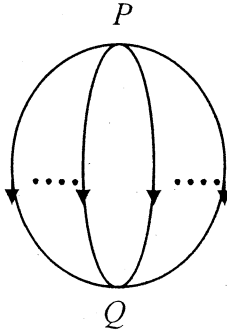
$$dP = \{dP, (q_1, 1), \dots, (q_r, 1)\}, \quad (5.7)$$

and so $|dP| = r + 1$. The thema dP has the following picture (meme).



2. Consider the following NAPM. If $v(P) = 1$, then

$$H_{w_{Q,v}} = \{(s_1, \mu_1), (s_2, \mu_2), \dots\}. \quad (5.8)$$



In (5.8), $\mu_i \in \mathbf{N}_+$, $s_i \in \mathbf{Q}$ and the ordered pair $s_i = (n_i, d_i)$, where $n_i \in \mathbf{Z}$ and $d_i \in \mathbf{N}_+$. The symbol i indexes the distinct weights s_i (not necessarily the distinct arrows) associated with the arrows in the multigraph. The n_i and d_i relatively prime. The number of pairs (s_i, μ_i) in the definition of $H_{w_{Q,v}}$ is not necessarily the same as the number of arrows in the graph. Then

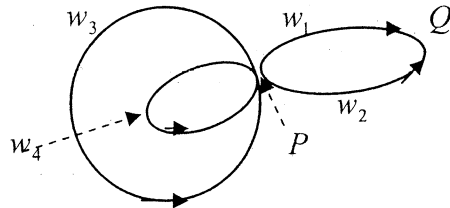
$$(s_i, \mu_i) = ((n_i, d_i), \mu_i) \quad n_i \in \mathbf{Z}, d_i \in \mathbf{N}_+, \mu_i \in \mathbf{N}_+. \quad (5.9)$$

That is, histograms of NAPMs are finite sets of ordered triples, i.e., are points in \mathbf{Z}^3 . The M-Z equations are

$$dP = \{dQ\} \quad \text{and} \quad dQ = H_{w_{Q,v}}. \quad (5.10)$$

Of course, the DLAPM is the indicated graph, labeled by the histograms and then decorated using the M-Z equations. Extending the concepts of Sect. 4, we take the resulting neural decoration as a meme with the set $\delta P = \{H_{w_{Q,v}}\}$ as its thema (the meme being a picture of the thema).

3. Next consider the following graph with two nodes, P and Q , where P is the point and where there are two arrows from P to P and two arrows from P to Q . The symbols w_1, \dots, w_4 denote the indicated synaptic weights.



With $\nu(P) = 1$, the M-Z equations for the corresponding DLAPG are

$$dP = \{dP, dQ\} \cup H_{P,v}, \quad (5.11)$$

$$dQ = H_{Q,v} = \begin{cases} \{(w_1,1), (w_2,1)\}, & w_1 \neq w_2 \\ \{(w_1,2)\}, & w_1 = w_2, \end{cases} \quad (5.12)$$

and

$$H_{P,v} = \begin{cases} \{(w_3,1), (w_4,1)\}, & w_3 \neq w_4 \\ \{(w_3,2)\}, & w_3 = w_4. \end{cases} \quad (5.13)$$

With distinct w_i , these equations imply

$$\begin{aligned} dP &= \{dP, H_{Q,v}\} \cup H_{P,v} \\ &= \{dP, \{(w_1,1), (w_2,1)\} \cup \{(w_3,1), (w_4,1)\}\} \\ &= \{dP, \{(w_1,1), (w_2,1)\}, (w_3,1), (w_4,1)\}. \end{aligned} \quad (5.14)$$

Note that $|dP| = 4$, and that the following three of its elements $\{(w_1,1), (w_2,1)\}$, $(w_3,1)$ and $(w_4,1)$ are distinct.

5.2 Special multigraphs

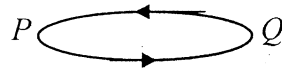
We have observed that in general a thema is not uniquely determined by a meme. A pointed multigraph (Γ, w, P) is called special if this uniqueness prevails. More precisely, consider the following definition.

Definition 5.3 (Special pointed multigraphs): A pointed multigraph (Γ, w, P) is said to be *special* if for any WF labeling λ of Γ , the thema (decoration of the point) $d_\lambda P$ uniquely determines the meme (the entire labeled decoration) d_λ .

We interpolate examples of special pointed multigraphs. Note that in these examples, expressions of the form $F(Q)$ for a node Q represent a generic WF label of the node Q .

Examples (con't.)

4.



The Aczel equations are

$$dP = \{dQ\} \cup F(P) \quad (5.15)$$

and

$$dQ = \{dP\} \cup F(Q). \quad (5.16)$$

Since both $F(P)$ and $F(Q)$ are WF, we have $F(P) = \mathcal{W}dP$, and so, $dP = \{dQ\} \cup \mathcal{W}dP$.

Then dQ is the only NWF element of dP . Now, $A - \mathcal{W}A = \{x \in A \mid x \text{ is NWF}\}$ for any $A \in S$. (See Sect 2.1 for the definition of \mathcal{W} .) Then applying this last relation to dP , we find

$$\{dQ\} = dP - \mathcal{W}dP. \quad (5.17)$$

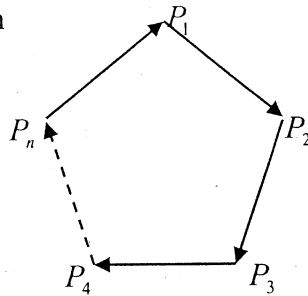
Then (5.14) may be written as⁴

$$dQ = \mathcal{U}(dP - \mathcal{W}dP), \quad (5.18)$$

since $\mathcal{U}\{A\} = A$ for any $A \in S$.

This example shows that for the graph in question we can recover the meme from the thema no matter what the labeling (with WF sets).

5. For the following graph



we have

$$dP_i = \{dP_{i+1}\} \cup F(P_i), \quad i = 1, \dots, n, \quad (5.19)$$

where $P_{n+1} \equiv P_1$. Next proceeding along the lines of Example 4, we have

$$dP_1 = \{dP_2\} \cup F(P_1) \quad (5.20)$$

and

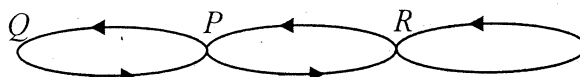
$$dP_2 = \mathcal{U}(dP_1 - \mathcal{W}dP_1). \quad (5.21)$$

Introducing the operator $\mathcal{V} = \mathcal{U}(I - \mathcal{W})$, dP_2 may be written as $dP_2 = \mathcal{V}dP_1$. This may be extended to give

$$dP_i = \mathcal{V}^{i-1}dP_1, \quad i = 2, \dots, n. \quad (5.22)$$

This shows that the entire meme corresponding the graph in this example (that is, all of the dP_i can each be written in terms of the thema dP_1 .

6. An example of a non-special graph is given for completeness. Namely,



⁴ The operator \mathcal{U} is defined as follows. $\mathcal{U}x = \{y \mid \exists z \in x \text{ such that } y \in z \in x\}$.

The Aczel equations are

$$dP = \{dQ, dR\} \cup F(P),$$

$$dQ = \{dP\} \cup F(Q)$$

and

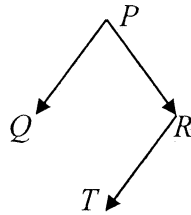
$$dR = \{dP, dR\} \cup F(R). \quad (5.23)$$

From the first of these we can deduce that the unordered pair $\{dQ, dR\}$ may be expressed in terms of the thema dP ,

$$\{dQ, dR\} = (I - \mathcal{W})dP. \quad (5.24)$$

Then with any solution pair $\{dQ, dR\}$, the pair $\{dR, dQ\}$ is also a solution.

7. The following diagram exhibits a well-founded graph that is not special.



The Aczel equations are

$$dP = \{dQ, dR\} \cup F(P),$$

$$dQ = F(Q),$$

$$dR = \{dT\} \cup F(R)$$

and

$$dT = F(T). \quad (5.25)$$

Then with any solution pair $\{dQ, dR\}$, the pair $\{dR, dQ\}$ is also a solution.

6. Non-Well-Founded Neural States.

In this section multigraphs are restricted so that the decoration of any node is NWF. This restriction enables development properties of reversibility and of self-awareness. In particular, (i) it allows for the retrieval of the M-Z histogram from the M-Z decoration and (ii) that corresponding to a particular so-called consciousness operator, an NAPM corresponding to a NWF multigraph is capable of self-awareness.

To begin we prescribe the non-well-foundedness of multigraphs (neural states).

Definition 6.1 (NWF multigraph, NWF neural state): A multigraph Γ is NWF if $\forall P \in N$, there exists an arrow whose source is P . A neural state (LAPM) is NWF if its underlying multigraph is NWF.

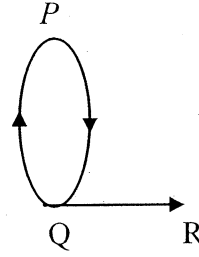
Recall that (a) for the weight function w , we have $w:A \rightarrow \mathbf{Q}$, (b) for a decoration d , we have $d:N \rightarrow \mathbf{S}$, while (c) for a histogram, we have $H:N \times \text{ran } w \rightarrow \mathbf{N}$, and that (d) the set

$$H(a) = \{(q, n) \mid q \in \mathbf{Q}, n \in \mathbf{N}_+, n \text{ is the frequency of } q \text{ as a value of } w|_{A_{a,v}}\} \quad (6.1)$$

is WF. However (e) the set $da = \{db \mid a \rightarrow b\} \cup H(a)$ may be NWF. We further recall that (f) $A_{a,v} = \{\text{arrows } \alpha \mid f(\alpha) = a, v(s(\alpha)) = 1\}$ and (g) $H(a) \subseteq (\text{ran } w) \times |N|$ is a finite set.

Examples

1. Consider the following graphs.



The M-Z equations for the graph on the left are

$$dP = \{dQ\} \cup H(P) \quad (6.2)$$

and

$$dQ = \{dP\} \cup H(Q). \quad (6.3)$$

From (6.3) we read off

$$dQ \in dP \in dQ \in dP, \quad (6.4)$$

which shows that dP and dQ are NWF. Indeed we may write

$$\mathcal{W}dP = H(P) \quad \text{and} \quad \mathcal{W}dQ = H(Q). \quad (6.5)$$

The M-Z equations for the graph on the right are

$$dP = \{dQ\} \cup H(P), \quad (6.6)$$

$$dQ = \{dR, dP\} \cup H(Q) \quad (6.7)$$

and

$$dR = H(R). \quad (6.8)$$

(6.6) and (6.7) give

$$dP \in dQ \in dP$$

and

$$dQ \in dP \in dQ. \quad (6.9)$$

So dP and dQ are NWF. However since histograms are WF, the third M-Z relation (6.8) shows that dR is WF. From these observations, we may write

$$\mathcal{W}dP = H(P)$$

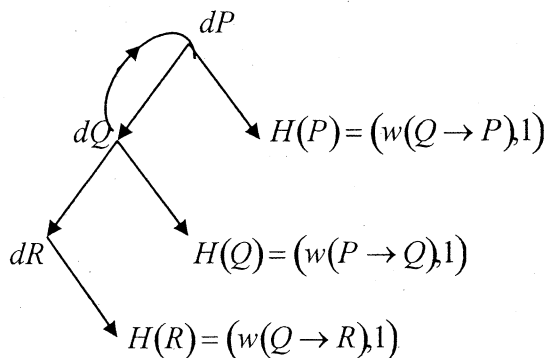
and

$$\mathcal{W}dQ = dR \cup H(Q). \quad (6.10)$$

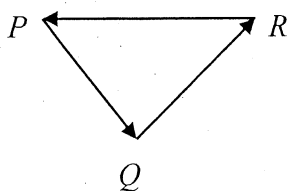
(6.6) and (6.7) give

$$dP = \{\{dR, dP\} \cup H(Q)\} \cup H(P). \quad (6.11)$$

All of this information enables us to compose the following picture of dP , where we have assumed that $v(P) = v(Q) = 1$.



2. The following diagram shows an example of a NWF graph with $|N| = |E| = 3$.



The M-Z equations are

$$dP = \{dQ\} \cup H(P),$$

$$dQ = \{dR\} \cup H(Q)$$

and

$$dR = \{dP\} \cup H(R). \quad (6.12)$$

We see that the sets dP , dQ and dR are NWF.

Retrieval of the histogram from the decoration: Returning to the general case, the concepts just developed allow us to retrieve the histogram from the decoration. This is the assertion of the following proposition.

Proposition 6.2: If Γ is finite and NWF, then for every node $a \in \Gamma$, da is NWF, and

$$da = \mathcal{W}(\{db|a \rightarrow b, db \text{ is WF}\} \cup H(a)) = H(a). \quad (6.13)$$

More generally,

Proposition 6.3: Let Γ be finite and NWF, and let λ be a labeling of Γ such that λa is WF for each $a \in N$. Then $d_\lambda a \in S_{nwf}$, and $\forall a \in N$, λa can be retrieved from $d_\lambda a$. In particular, $\lambda a = \mathcal{W}d_\lambda a$.

Consciousness operators: As indicated in Fig 1.1, there is a class of operators called consciousness operators (denoted generically as \mathcal{K}) that play a fundamental role in the foundations developed in M-Z. From Fig 1.1, we see that conscious awareness resides in the Platonic realm. The Russell operator \mathcal{R} and the well-founded-part-of operator \mathcal{W} (Def 2.3) are examples of \mathcal{K} . (See the Appendix for a proof as well as for a fundamental characterization of \mathcal{W}). We can apply Prop. 6.2 to aspects of the structure in Fig.1.1 to frame an example of awareness that resides in the physical realm. We interpret these observations as follows.

Observation 6.4: A NWF NAPM is capable of self-awareness.

This can be deduced from the diagram in Fig. 6.5, since the histogram is physical.

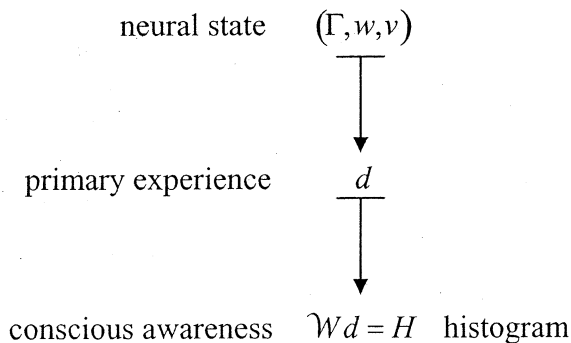


Figure 6.5: The histogram of a neural state as awareness

We claim that Obs. 6.4 addresses the following critique concerning awareness and the mind in science.

“A physical scientist does not introduce awareness (sensation or perception) into his theories, and having thus removed the mind from nature he cannot expect to find it there.”

Schrödinger, 1958

PART II: DYNAMICS

7. Voltage, Activity and Histogram Dynamics

In this section we show how the Mc-P equations that describe the voltage dynamics on a neural net induce a corresponding dynamics for the associated histograms. That is, we derive what is called the histogram evolution equation. A byproduct is the introduction of a notion of nodal activity and its associated dynamics.

We begin with a statement of those (discrete time) Mc-P dynamics (Haykin, 2008), which specifies the input-output voltage relationship for a model neuron. (See (2.5).)

$$\begin{aligned} \text{Voltage dynamics: } v(a, t+1) &= h \left(\left(\sum_{\{\alpha | f(\alpha)=a\}} w(\alpha) v(s(\alpha), t) \right) - \theta \right) \\ &\stackrel{\text{def}}{=} h_{\theta} \left(\sum_{\{\alpha | f(\alpha)=a\}} w(\alpha) v(s(\alpha), t) \right). \end{aligned} \quad (7.1)$$

In (7.1) α runs over those arrows in A for which $f(\alpha) = a$, t is an integer valued time, h the Heaviside function and θ the neuronal firing threshold. For clarity we take the synaptic weights $w(\alpha)$ for each arrow in Γ to be time independent.

The Mc-P equation motivates introduction of a construct called nodal activity:

Definition 7.1 (Nodal activity): We call $G(a)$ the activity at the node $a \in N$, where

$$\begin{aligned} G(a) &= \sum_{\{\alpha | f(\alpha)=a\}} w(\alpha) v(s(\alpha)). \\ &\stackrel{\text{def}}{=} G_{\Gamma, w}(v)(a). \end{aligned} \quad (7.2)$$

So $G_{\Gamma, w}$ is a mapping $V_N \rightarrow \mathbf{Q}^N$. Combining (7.1) and (7.2) gives

$$v(a, t+1) = h_{\theta}(G(a, t)). \quad (7.3)$$

Then combining (7.2) and (7.3) produces a specification of activity dynamics. Namely,

$$\text{Activity dynamics: } G(a, t+1) = \sum_{\{\alpha | f(\alpha)=a\}} w(\alpha) h_{\theta}(G(s(\alpha), t)) \quad (7.4)$$

Using (7.2) and (7.3), we deduce the following proposition.

Proposition 7.2: The voltage trajectory determines the activity trajectory and conversely.

As the voltage varies with time, so will the corresponding histograms. With the latter, we can use the M-Z equation to decorate the graph (or multigraph) as time evolves. That is, we can determine the dynamics for the associated memes and themata. (See Sect. 4.) We now develop an expression for the histogram dynamics.

Letting κ_σ denote what we shall call the Kronecker function, namely the function whose values are specified by $\kappa_\sigma(\mu) = \delta_\sigma$, where δ_σ is the Kronecker- δ . Then we may write the following μ independent representation of $H(a, t)$, the histogram at node a at time t . (See Def. 5.1.)

$$\begin{aligned} H(a, t) &= \sum_{\alpha \in A_{a,v}} \kappa_{w(\alpha)} \\ &= \sum_{\{\alpha | f(\alpha) = a\}} v(s(\alpha), t) \kappa_{w(\alpha)}. \end{aligned} \quad (7.5)$$

Recall that the argument of $\kappa_{w(\alpha)}(\mu)$ is restricted to be rational ($\mu \in \mathbf{Q}$).

Increasing t to $t+1$ in (7.5) and then using the Mc-P dynamics in (7.1), we obtain

$$\begin{aligned} H(a, t+1) &= \sum_{\{\alpha | f(\alpha) = a\}} v(s(\alpha), t+1) \kappa_{w(\alpha)} \\ &= \sum_{\{\alpha | f(\alpha) = a\}} h_\theta \left(\sum_{f(\beta) = s(\alpha)} w(\beta) v(s(\beta), t) \right) \kappa_{w(\alpha)} \\ &= \sum_{\{\alpha | f(\alpha) = a\}} h_\theta \left(\sum_{\mu} \mu H(s(\alpha), t)(\mu) \right) \kappa_{w(\alpha)}, \end{aligned} \quad (7.6)$$

where (7.5) is used for deducing the last line in (7.6).

Here and hereafter, the summation variable μ runs over $\text{ran } w$, a finite subset of \mathbf{Q} . The interior sum in the last expression in (7.6) has the form of an inner product that we write as

$$\sum_{\mu} \mu F(\mu) = \langle L, F \rangle, \quad (7.7)$$

where $L(\mu) = \mu$ and $F = H(s(\alpha), t)$. Then a preliminary form of the histogram dynamics are given by

$$H(a, t+1) = \sum_{\{\alpha | f(\alpha) = a\}} h_\theta \left(\langle L, H(s(\alpha), t) \rangle \right) \kappa_{w(\alpha)}. \quad (7.8)$$

Noting that $H : N \times \mathbf{Z} \times \mathbf{Q} \rightarrow \mathbf{N}$, we rewrite the histogram value $H(a, t)(\mu)$ as $H(a, t, \mu)$. Then we also rewrite the equation (7.5) for the histogram as

$$\begin{aligned}
H(a,t,\mu) &= \sum_{\{\alpha|f(\alpha)=a\}} v(s(\alpha),t) \begin{cases} 1, & w(\alpha)=\mu \\ 0, & w(\alpha)\neq\mu \end{cases} \\
&= \sum_{\{\alpha|f(\alpha)=a,w(\alpha)=\mu\}} v(s(\alpha),t). \tag{7.9}
\end{aligned}$$

This last sum takes its values in \mathbf{N} , since it is a sum of zeros and ones. Rewrite (7.8) as

$$H(a,t+1,\mu) = \sum_{\{\alpha|f(\alpha)=a,w(\alpha)=\mu\}} h_\theta \left(\langle L, H(s(\alpha),t) \rangle \right). \tag{7.10}$$

Combining (7.7) with (7.10) we get the equation specifying the histogram dynamics. Namely,

$$\textbf{Histogram dynamics: } H(a,t+1,\mu) = \sum_{\{\alpha|f(\alpha)=a,w(\alpha)=\mu\}} h_\theta \left(\sum_{\lambda} \lambda H(s(\alpha),t,\lambda) \right). \tag{7.11}$$

Now note that

$$\begin{aligned}
\sum_{\lambda \in w(A)} \lambda H(b,t,\lambda) &= \sum_{\lambda \in w(A)} \lambda \sum_{\{\alpha|f(\alpha)=a,w(\alpha)=\lambda\}} v(s(\alpha),t) \\
&= \sum_{\{\alpha|f(\alpha)=b\}} w(\alpha) v(s(\alpha),t) \\
&= G(b,t), \tag{7.12}
\end{aligned}$$

the last following from (7.4). Combining (7.11) and (7.12) gives

$$H(a,t+1,\mu) = \sum_{\{\alpha|f(\alpha)=a,w(\alpha)=\mu\}} h_\theta \left(G(s(\alpha),t) \right). \tag{7.13}$$

The relationships between the voltage, activity and histogram trajectories is specified in the following Prop. 7.3 and Thm. 7.4. Prop. 7.3 follows from (7.12) and (7.13).

Proposition 7.3: The histogram trajectory determines the activity trajectory and conversely.

Combining Prop. 7.1 and Prop. 7.2 yields the following theorem.

Theorem 7.4: Any one of the three trajectories (voltage, activity and histogram) determines the remaining two.

We conclude Sect. 7 with the following expository observations.

Remark 7.5: To use the histogram dynamics, we must specify an initial histogram, $H(a,0,\mu)$. This can be determined from the weights and a specification of the initial value of the voltage (zero or one) at each node of Γ . Finally consider the evolution of the corresponding DLAPMs. As the net evolves, the corresponding DLAPMs evolve, and so the associated thema (the set decorating the point of Γ) evolves along with them. Note that even though the weights are frozen, the voltages evolve, and so, the time development of the histogram may be different for different specifications of the initial voltage values. In this way the temporal development of the neural net being modeled may instantiate a number of different thema trajectories. That is, a neural net with frozen weights may manifest one of a finite set of different evolving states of mind (see Table 2.2). This description is compatible with the customary role of a neural net as an associative memory. A neural net can store a (finite) collection of different information records in terms of its weights. The particular memory record that is delivered by the Mc-P dynamics by accessing the net with voltage input cues depends on those inputs.

8. Crop Dynamics and Mean Dynamics

In order to conceptualize the set of all M-Z histograms, we introduce the auxiliary notion of a crop. A crop is an abstraction of an M-Z histogram that eliminates the need to specify such details as weight and voltage data. Then a specification of crop dynamics that emerges as a natural generalization of histogram dynamics is given. The notion of the mean of a crop is introduced, and the relationship of its dynamics to the crop dynamics is developed.

To begin, fix the multigraph $\Gamma = (N, A)$ and the weight function $w : A \rightarrow \mathbf{Q}$. Note that the range of weights $w(A)$ is a finite subset of \mathbf{Q} . Note also that there is a corresponding WF finite set of possible histograms associated with Γ and w , which we write as follows.

$$(1 + \deg f)^{w(A)}, \quad (8.1)$$

where $\deg f = \max_{a \in N} |f^{-1}(a)|$.

Next we introduce a set $F(a)$, an abstraction of a histogram. $F(a)$ is chosen so that $F(a) \in (1 + \deg f)^{w(A)}$. Note that $F(a)$ is a finite WF set. Let us alter the specification of the function $d : N \rightarrow \mathcal{S}$, by replacing $H(a)$ with $F(a)$ and using the following relation (compare (5.2)).

$$da = \{db | a \rightarrow b\} \cup F(a), \quad \forall a \in N, \quad (8.2)$$

where

$$F : N \rightarrow (1 + \deg f)^{w(A)}. \quad (8.3)$$

We make use of the notions of a crop on a set and a crop space specified as follows.

Definition 8.2 (Crop, crop space): A crop on a set Y is any function $f : Y \rightarrow \mathbf{N}$. A crop space is a collection of such functions. Given a multigraph supplied with a weight function, (Γ, w) , the crop space, $C_{\Gamma, w}$, associated with (Γ, w) is given by

$$\begin{aligned} C_{\Gamma, w} &= (1 + \deg f)^{w(A) \times \mathbf{N}} \\ &= \left((1 + \deg f)^{w(A)} \right)^{\mathbf{N}}. \end{aligned} \quad (8.4)$$

Since $|V_N| = 2^{|\mathbf{M}|}$ and $|C_{\Gamma, w}| = (1 + \deg f)^{w(A) \times |\mathbf{N}|}$, crop space is typically larger than voltage space. The two are of the same size in the special case that $\deg f = |w(A)| = 1$.

The form of the histogram dynamics in (7.11) suggests that the following dynamics be taken for a function $F \in C_{\Gamma, w}$. (Recall that F is a generalization of a histogram.)

Crop dynamics:
$$F(a, t + 1, \mu) = \sum_{\{\alpha | f(\alpha) = a, w(\alpha) = \mu\}} h_\theta \left(\sum_{\lambda} \lambda F(s(\alpha), t, \lambda) \right). \quad (8.5)$$

In the non-degenerate case that $w : A \rightarrow \mathbf{Q}$ is 1-1 (that is, when all arrows have different weights), the expression for the crop dynamics in (8.5) simplifies to

$$F(a, t + 1, \mu) = \delta_{a, f(w^{-1}(\mu))} h_\theta \left(\sum_{\lambda \in w(A)} \lambda F(s(w^{-1}(\mu)), t, \lambda) \right). \quad (8.6)$$

Now consider the construct called the mean (of a crop) specified as follows.

Definition 8.3 (Mean of a crop): The mean of a crop $M(b) \in \mathbf{Q}^{\mathbf{N}}$ is given by

$$\begin{aligned} M(b) &= \sum_{\lambda \in w(A)} \lambda F(b, \lambda) \\ &= \mathbb{E} M(F)(b). \end{aligned} \quad (8.7)$$

We regard M as a mapping $C_{\Gamma, w} \rightarrow \mathbf{Q}^{\mathbf{N}}$. Next combining (8.5) with (8.7) gives

$$F(a, t + 1, \mu) = \sum_{\{\alpha | f(\alpha) = a, w(\alpha) = \mu\}} h_\theta (M(s(\alpha), t)). \quad (8.8)$$

Combining (8.7) and (8.8) gives an equation for the mean dynamics. Namely,

Mean dynamics:
$$M(a, t + 1) = \sum_{\{\alpha | f(\alpha) = a\}} w(\alpha) h_\theta (M(s(\alpha), t)). \quad (8.9)$$

The following proposition follows from (8.7) and (8.9). (Compare with Prop. 7.3.)

Proposition 8.4: A crop trajectory determines a mean trajectory and conversely.

Now using (8.7), rewrite (7.12) as

$$G(t) = MH(t). \quad (8.10)$$

We can retrieve the voltage from the histogram by combining (8.10) with (7.3). Namely

$$v(t+1) = h_\theta MH(t). \quad (8.11)$$

This *demonstrates the reversibility* of the M-Z Theory.

9. The Aczel Transform and Memetic Dynamics

We now complete the layering of dynamical systems (all of which are autonomous) defined in terms of a weighted multigraph (Γ, w) by specifying decoration dynamics. In order to introduce decoration dynamics, we specify a new tool \mathcal{A}_Γ called the Aczel transform that for a given Γ associates d to F as follows.

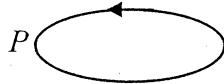
Definition 9.1 (Aczel transform): Let $F \in S^N$, where S^N denotes the class of all set valued functions on N . Then with d specified in (8.2), the Aczel transform \mathcal{A}_Γ is given by

$$F \mapsto \mathcal{A}_\Gamma F = d. \quad (9.1)$$

Note that (2.8), which expresses the M-Z Theory can be written in terms of \mathcal{A}_Γ as follows.

$$d = \mathcal{A}_\Gamma H. \quad (9.2)$$

To illustrate these constructs consider the case $\Gamma = (P, P \rightarrow P)$ displayed as follows.



Then

$$d(P) = (\mathcal{A}_\Gamma F)(P) = \{ \mathcal{A}_\Gamma F(P) \} \cup F(P), \quad (9.3)$$

where $F(P) \in S$, and $F(P) \mapsto (\mathcal{A}_\Gamma F)(P)$ is a transformation on sets. If $F(P) = \{x\}$, a singleton, then

$$\begin{aligned} d(P) &= \{d(P)\} \cup \{x\} \\ &= \{d(P), x\} \\ &= Dx. \end{aligned} \quad (9.4)$$

(See footnote 3 for the definition of \mathcal{D} .) The following hypothesis that abstracts features of biological neural networks plays a critical role in the remaining development.

Hypothesis 9.2: We hereafter assume that Γ is finite and NWF. (See Def. 6.1.)

We can now state the following proposition and corollary that characterize several reciprocal properties of our constructs.

Proposition 9.3: Suppose $F : N \rightarrow S_{wf}$. Then

$$d : N \rightarrow S_{nwf}, \quad (9.5)$$

and

$$\mathcal{W}d = F, \quad (9.6)$$

showing that $F(a)$ can be recovered from the decoration d .

The following corollary follows from (8.2) and (9.4).

Corollary 9.4
$$da = \{db \mid a \rightarrow b\} \cup \mathcal{W}da. \quad (9.7)$$

Next we specify the construct of decoration space, employing the following notation. For O any operator and for any set $B \subseteq \text{dom } O$, let

$$O[B] = \{Ob \mid b \in B\} = \text{ran } O|_B. \quad (9.8)$$

Definition 9.5 (Decoration space): The set $\mathcal{A}_\Gamma[C_{\Gamma,w}]$ is called decoration space associated to (Γ, w) .

We continue by assembling three of the layers of the dynamics already available.

A) Voltage (Mc-P) dynamics: The collection of voltages associated with (Γ, w) , namely $\{0,1\}^N$ is what we call the voltage space V_N . We symbolize the dynamics on this space by introducing a voltage dynamics operator $E_{\Gamma,w}$, rewriting (7.1) as

$$v(t+1) = E_{\Gamma,w}v(t). \quad (9.9)$$

B) Crop dynamics and Histogram dynamics: We introduce a crop dynamics operator $T_{\Gamma,w,\theta}$ in terms of which we symbolize the crop dynamics of (8.5), (8.6):

$$F(t+1) = T_{\Gamma,w,\theta}F(t). \quad (9.10)$$

Note that $T_{\Gamma,w,\theta}$ is an operator on set valued functions on N . Note also that crop dynamics are intrinsically determined by the weighted multigraph (Γ, w) .

We now introduce a mapping $U_{\Gamma,w}$ that relates voltage data and crop data. Namely,

$$U_{\Gamma,w}(v)(a,\mu) = \left\{ \alpha \mid v(s(\alpha)) = 1, f(\alpha) = a, w(\alpha) = \mu \right\}. \quad (9.11)$$

Since a histogram is a special case of a crop, we have in particular, that

$$H = U_{\Gamma,w}v. \quad (9.12)$$

For any voltage v , $U_{\Gamma,w}$ takes values in the set $1 + \text{deg } t$. So $U_{\Gamma,w} \in C_{\Gamma,w}$. Indeed the mapping $U_{\Gamma,w} : V_N \rightarrow C_{\Gamma,w}$ gives the frequency of μ for $w|_{A_{a,v}}$.

For clarity we give an operator form of histogram dynamics obtained from (9.10) by specializing F to H as follows.

$$H(a,t+1,\mu) = (T_{\Gamma,w,\theta}H(t))(a,\mu). \quad (9.13)$$

C) Decoration dynamics: The Aczel transform \mathcal{A}_Γ and the crop dynamics operator $T_{\Gamma,w,\theta}$ along with Prop. 9.3 enable introduction of a dynamics for decorations. In particular,

$$d(t+1) = S_{\Gamma,w,\theta}d(t), \quad (9.14)$$

where the decoration dynamics operator $S_{\Gamma,w,\theta}$ is specified as follows.

$$S_{\Gamma,w,\theta} = \mathcal{A}_\Gamma T_{\Gamma,w,\theta} \mathcal{A}_\Gamma^{-1}. \quad (9.15)$$

Using this with (9.6), and then using $F = \mathcal{A}_\Gamma^{-1}d$, the last deducible from (9.1), we find

$$S_{\Gamma,w,\theta}d = \mathcal{A}_\Gamma T_{\Gamma,w,\theta} \mathcal{W}d, \quad (9.16)$$

where d is specified in (8.2). Finally using (9.13) and (9.15), we obtain the following operator form of the decoration dynamics.

$$d(a,t+1) = (\mathcal{A}_\Gamma T_{\Gamma,w,\theta} \mathcal{W}d(t))(a). \quad (9.17)$$

For the definitions of \mathcal{A}_Γ , $T_{\Gamma,w,\theta}$ and \mathcal{W} , see (9.1), (9.10) and (2.5), respectively. Note that \mathcal{W} is a node-wise operator, while \mathcal{A}_Γ and $T_{\Gamma,w,\theta}$ are operators on functions.

The developments in this section are summarized in Fig. 9.6.

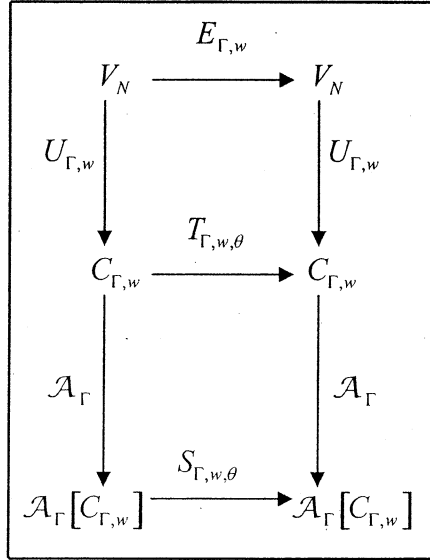


Figure 9.6: Voltage, crop and decoration dynamics

10. Configuration Spaces (Statics)

We now complete the complement of spaces needed for our development by introducing both a space that represents all of the memes associated with a given (Γ, w) and a space that represents all of the themata associated with a given (Γ, w, P) .

First recall that $U_{\Gamma,w}$ (see (9.11)) is a mapping of voltage space into crop space (Def. 8.2).

$$U_{\Gamma,w} : V_N \rightarrow C_{\Gamma,w}. \quad (10.1)$$

The pair $(\Gamma, U_{\Gamma,w}(v))$ constitutes a labeled multigraph. We continue with the introduction of five additional constructs.

Definition 10.1 (Mapping of a voltage function to a meme): The mapping $D_{\Gamma,w}$ taking a voltage function into a meme is given by

$$D_{\Gamma,w}(v) = \mathcal{A}_\Gamma(U_{\Gamma,w}(v)). \quad (10.2)$$

We shall write

$$D_{\Gamma,w} = \mathcal{A}_\Gamma U_{\Gamma,w}, \quad (10.3)$$

so that

$$d(t) = D_{\Gamma,w}v(t). \quad (10.4)$$

The following definition is framed in terms of this construct.

Definition 10.2 (Memetic space): Memetic space, $Me_{\Gamma,w} \in S$ is defined by the following relations. (See (9.8).)

$$\begin{aligned} Me_{\Gamma,w} &= D_{\Gamma,w}[V_N] \\ &= \mathcal{A}_{\Gamma} U_{\Gamma,w}[V_N]. \end{aligned} \quad (10.5)$$

Next recall that δ maps neural states into S . In particular, δ takes a DLAPM into the decoration of the point P of the APM. Then we write (see (2.4))

$$\delta(\Gamma, w, v, P) = D_{\Gamma,w}(v)P. \quad (10.6)$$

We define thematic space as follows.

Definition 10.3 (Thematic space): Thematic space $Th_{\Gamma,w,P} \in S$ is given by

$$\begin{aligned} Th_{\Gamma,w,P} &= \delta_{\Gamma,w,P}[V_N] \\ &= \{ \delta(\Gamma, w, v, P) | v \in V_N \}. \end{aligned} \quad (10.7)$$

We next define $R_{\Gamma,w,P}$ to be interpreted as a realization mapping, a mapping that takes an unconscious neural state into a state of mind (see Table 2.2). That is, a mapping that takes a meme into a thema.

Definition 10.4 (Realization mapping): The realization mapping $R_{\Gamma,w,P}$, taking memetic space $Me_{\Gamma,w}$ into thematic space $Th_{\Gamma,w,P}$, is specified as follows.

$$R_{\Gamma,w,P} : D_{\Gamma,w}[V_N] \rightarrow \delta_{\Gamma,w,P}[V_N], \quad (10.8)$$

where

$$D_{\Gamma,w}(v) \mapsto D_{\Gamma,w}(v)P. \quad (10.9)$$

Finally we introduce a notion called a summit that generalizes a thema.

Definition 10.5 (Summit space): The construct $\delta_{\Gamma,w,P} \mathcal{A}_{\Gamma}[C_{\Gamma,w}]$ is given the name summit space and its elements are called summits.

The diagram in Fig. 10.7 shows relationships among our configuration spaces and mappings. The constructs in the right column of the diagram are mathematical abstractions that are more general than those in the left column, which in particular, also depend on the voltage. The horizontal arrows (\hookrightarrow) represent the inclusion mappings, I_{his} , I_{mem} and I_{them} , respectively. These three injections are well defined since every histogram is a crop, every meme is a decoration and every thema is a summit. The dashed line separates sets in the diagram that are WF from those that are NWF.

The following theorem characterizes key properties of these constructs and mappings.

Theorem 10.8: The diagram of configuration spaces and mappings in Fig. 10.7 is a commutative diagram.

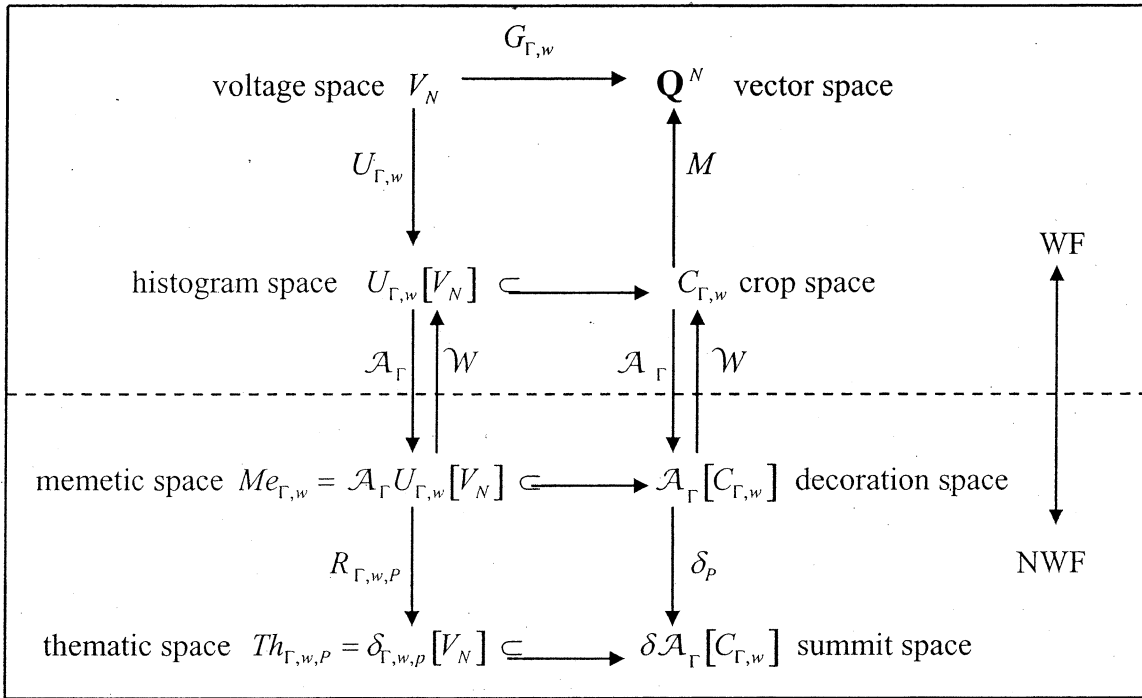


Figure 10.7: Relations among configuration spaces and mappings

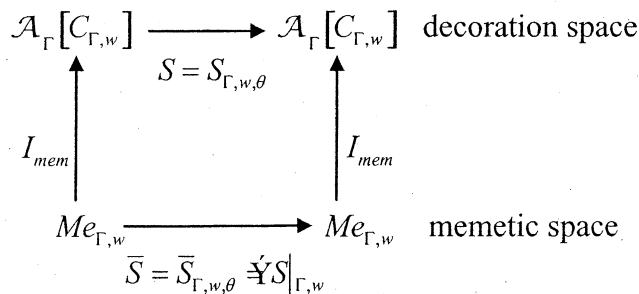
11. Configuration Spaces (dynamics)

We shall now introduce a well-defined dynamics for memes. The relationship of these dynamics to those already introduced (for voltage, crop and decoration) is displayed in a three dimensional diagram shown in Fig. 11.2. Following this the associated dynamics for themata (although not uniquely defined) are specified.

A) Memetic dynamics: Let

$$\bar{S} = \bar{S}_{\Gamma, w, \theta} \ni S|_{\Gamma, w, \theta} \quad (11.1)$$

denote the memetic dynamics operator expressed in terms of the decoration dynamics operator specified by (9.12) - (9.14). This operator is defined by the following diagram.



Since a meme is a decoration, (9.17) is an explicit equation for memetic dynamics.

To illustrate the constructs thus far introduced, we combine this diagram with the one in Fig. 9.6. The resulting diagram of the relationships of the dynamical systems of our theory is shown in Fig 11.2, where for clarity, we have omitted most subscripts.

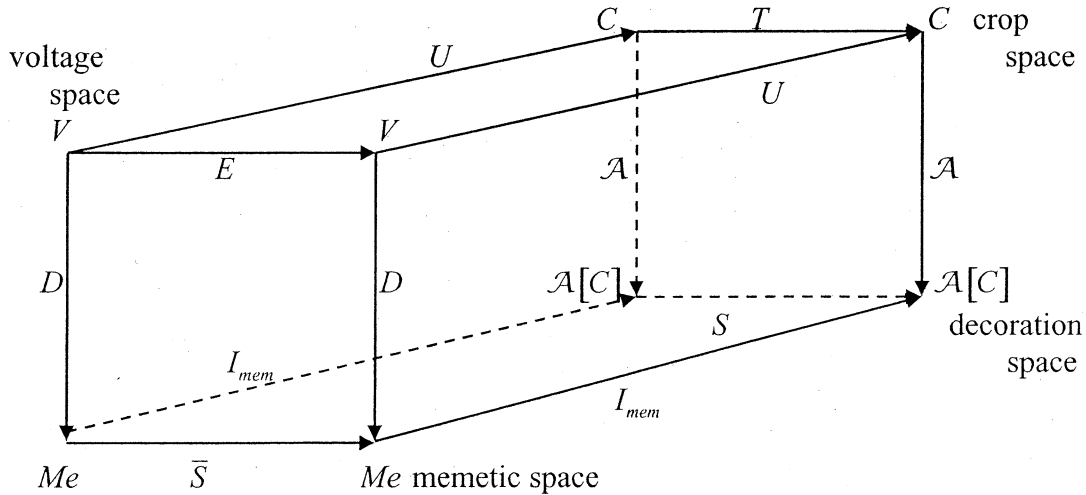


Figure 11.2: Diagram of the dynamical systems

The following Theorem 11.3 characterizes key interrelationship properties of the constructs in the diagram in Fig. 11.2.

Theorem 11.3: The diagram of dynamical systems in Fig 11.2 is a commutative diagram.

B) Thematic dynamics: The realization mapping of (10.8) takes a meme μ into its thema τ , say. That is,

$$\tau = R_{\Gamma,w,p}\mu. \quad (11.2)$$

More than one μ , albeit a finite number, may satisfy (11.2). Each choice of μ evolves to give a memetic trajectory, $\mu_n = \bar{S}^n \mu$, $n = 0, 1, \dots$, generating thereby an autonomous thematic trajectory. Namely,

$$\tau_n = R_{\Gamma,w,p} \bar{S}^n \mu, \quad n = 0, 1, \dots \quad (11.3)$$

In fact, given the value of the thema τ_n , $n = 0, 1, \dots$, μ may be replaced by any meme μ_n that satisfies the equation $\tau_n = R_{\Gamma,w,p} \mu_n$. This procedure generates a non-autonomous trajectory. Namely,

$$\tau_n = R_{\Gamma,w,p} \bar{S}^n \mu_n, \quad n = 0, 1, \dots \quad (11.4)$$

12. Summary and Semantics

The foundations of consciousness developed in M-Z emanated from a framework of mappings associated with the M-Z Theory. These mappings describe a static picture,

showing an irreversible passage from the physical to the Platonic realm. That is, the physical cannot be recovered from the ideal (Platonic) quality that is consciousness. The introduction of dynamics introduces reversibility into our study, and so, it gives a more detailed description of the relationship between physical and Platonic realms of mental activity. The multigraph Γ being NWF (recall Hyp. 9.2) is a critical requirement for reversibility. We start in Sect. 12.1 with a syntactic and semantic summary of the physical aspects of the dynamical developments and conclude in Sect. 12.2 with a discussion of the semantics associated with the Platonic aspects of our constructs.

12.1 Review of the constructs

Dynamical variables: In Table 12.1 the semantics of the physical variables, $v(t)$, $G(t)$, $H(t)$ as well as the Platonic variable $d(t)$ are displayed.

	Semantics	Takes values in
v	voltage	$\{0,1\}$
G	activity	\mathbf{Q}
H	intrinsic data	\mathbf{N}^q (crops on \mathbf{Q})
d	meme	NWF sets

Table 12.1: Semantics of the dynamic variables

Equations of the dynamics: These equations, specified in (7.1), (7.3), (7.11), (9.13) and (9.17) are listed here.

$$\text{Voltage:} \quad v(a, t+1) = h_\theta \left(\sum_{\{\alpha | f(\alpha)=a\}} w(\alpha) v(s(\alpha), t) \right). \quad (7.1)$$

$$\text{Activity:} \quad G(a, t+1) = \sum_{\{\alpha | f(\alpha)=a\}} w(\alpha) h_\theta (G(s(\alpha), t)). \quad (7.3)$$

$$\text{Histogram:} \quad H(a, t+1, \mu) = \sum_{\{\alpha | f(\alpha)=a, w(\alpha)=\mu\}} h_\theta \left(\sum_\lambda \lambda H(s(\alpha), t, \lambda) \right). \quad (7.11)$$

$$= (T_{\Gamma, w, \theta} H(t))(a, \mu). \quad (9.13)$$

$$\text{Memetic:} \quad d(a, t+1) = (\mathcal{A}_\Gamma T_{\Gamma, w, \theta} \mathcal{W} d(t))(a). \quad (9.17)$$

Temporal structure: The dynamics allows us to express v in terms of H or G . The temporal structure of the M-Z Theory is summarized in Table 12.2, where the static/dynamic type (S/D) and semantics of the syntactical constituents are displayed. The column of references indicates equation numbers in the text.

	Syntax	Type	Semantics	Ref
1	$G(t) = \sum_{\{\alpha f(\alpha)=a\}} w(\alpha)v(s(\alpha),t)$	S	Definition of neural activity	(7.2)
2	$v(t+1) = h_\theta(G(t))$	D	Mc-P dynamics	(7.4)
3	$G(t) = MH(t)$	S	Histogram \rightarrow Activity	(8.8)
4	$H(a,t+1,\mu) = \sum_{\{\alpha f(\alpha)=a.w(\alpha)=\mu\}} h_\theta(G(s(\alpha),t))$	D	Activity \rightarrow Histogram	(7.13)
5	$H(t) = U_{\Gamma,w}v(t)$	S	M-Z Theory	(9.12)
6	$v(t+1) = h_\theta MH(t)$	D	Reversibility of M-Z theory	(8.11)
7	$d(t) = \mathcal{A}_\Gamma H(t)$	S	M-Z Theory	(9.2)
8	$H(t) = \mathcal{W}d(t)$	S	Reversibility of \mathcal{A}_Γ	(6.5)
9	$d(t) = D_{\Gamma,w}v(t)$	S	M-Z Theory	(10.3)

Table 12.2: Temporal structure of the M-Z Theory

Mappings: The mappings in Table 12.2 that display interrelationships of our constructs are assembled in a reversible portion of a diagram of mental activity in Fig. 12.3.

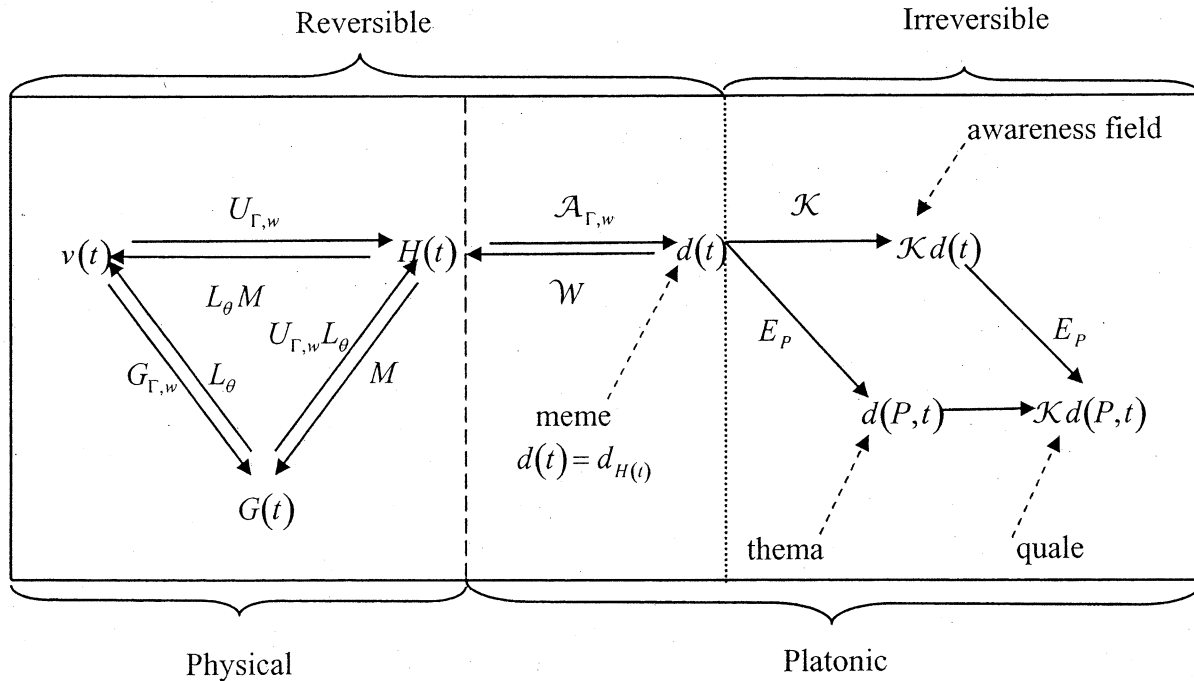


Figure 12.3: Summary of constructs and mappings of mental activity, indicating the partitioning into reversible/irreversible features and physical/Platonic features, resp.

All mappings in Fig. 12.3 are static except for L_θ , which is defined in terms of h_θ as follows.

$$(L_\theta f)(t) = h_\theta(f(t-1)). \quad (12.1)$$

The three constructs $v(t)$, $G(t)$ and $H(t)$ and the six mappings $G_{\Gamma,w}$, $U_{\Gamma,w}$, L_θ , $U_{\Gamma,w}L_\theta$, M and $L_\theta M$ appear in the triangle on the left in Fig. 12.3. The paired arrows express the reversibility. A dashed line separates the physical realm from the Platonic. The middle part of the diagram portrays the reversible passage between $H(t)$ and $d(t)$. The right hand part of the diagram (separated from the rest by a dotted line) shows the Platonic constructs from which, because of a loss of information in their formulation, the physical cannot in general be recovered. (Do recall from Sect. 5.2 however, that the meme may be recovered from the thema in the case of so-called special multigraphs.) Also shown in the right hand part is \mathcal{K} a generic consciousness operator (see the Appendix) as well as the mapping $E_p : S^N \rightarrow S$, where

$$E_p d = dP. \quad (12.2)$$

12.2 Semantics of awareness

Some of the constructs shown in the Platonic portion of Fig. 12.3 (see Fig. 1.1 also) are labeled with semantic descriptors that connect the syntactical constructs of our development to one or another of the philosophical aspects of consciousness studies. We list these along with comments as follows.

Meme: A meme is a concept encoded physically in terms of synaptic weights. It arises by application of $D_{\Gamma,w}$ (entry 9 in Table 12.2), so we view it as an unconscious aspect of a mental state. The concept corresponds to a DLAPM, and so, the meme could be viewed as a field of unconscious primary experience (see Fig. 1.1) spread over a neural network.

Thema: The thema (Def. 4.2) is a primary experience (see Fig. 1.1), a preconscious feature that can be viewed as the literal theme of a meme.

Consciousness operator: A consciousness operator (see M-Z and the axioms in the Appendix) is a mapping that takes (a) a meme into an awareness field or (b) a thema into conscious awareness.

Awareness field: Awareness is a conscious experience. An awareness field is a field of conscious awareness spread over a neural network. It is the collection of results obtained by applying a consciousness operator to a meme (the collection of sets comprising a decoration).

Quale: A quale is a conscious manifestation such as a color, sound, wetness... or a feeling such as hunger, pain, longing... It is the perceptual experience that we identify as consciousness. A quale is an example of a conscious awareness derived from a thema.

Appendix

Intrinsic fundamental properties of \mathcal{W} are the subject of the following remark.

Remark: 1. For every consciousness operator \mathcal{K} , $\mathcal{W} \subseteq \mathcal{K}$.

2. \mathcal{W} is the unique consciousness operator whose range consists of WF sets.

To demonstrate that an operator is a consciousness operator, we must show that it satisfies the following four consciousness operator axioms in M-Z.

	Axiom	Semantic interpretation of the axiom	Name of Axiom
a)	$\forall x, \mathcal{K}x \subseteq x$	Experience generates its own awareness	Generation
b)	$\forall x, x \notin \mathcal{K}x$	Awareness does not generate the primary experience	Irreversibility
c)	$\forall x, \mathcal{K}x \not\subseteq x$	Awareness is removed from experience	Removal
d)	If $x \subseteq y$, then $\mathcal{K}x = x \cap \mathcal{K}y$	Awareness of a sub-experience is determined by the sub-experience and awareness of the primary experience	Selection

Table A1: The axioms for a consciousness operator.

Such a demonstration for \mathcal{R} is found in M-Z. The demonstration for \mathcal{W} is straight forward, but for the convenience of the reader, we shall show b) and c).

b) Since $\mathcal{W}x \subset \mathcal{R}x$, we deduce that $x \in \mathcal{W}x \Rightarrow x \in \mathcal{R}x$, a contradiction

c) Now suppose $\mathcal{W}x \in x$. Then either 1) $\mathcal{W}x \in \mathcal{W}x$ or 2) $\mathcal{W}x \in x - \mathcal{W}x$.

1) $\mathcal{W}x \in \mathcal{W}x \Rightarrow \mathcal{W}x$ is NWF, which contradicts Lemma 6.4.

2) $\mathcal{W}x \in x - \mathcal{W}x$. Then by definition $\mathcal{W}x$ is NWF, contradicting Lemma 6.4.

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