The Renormalization of Information

Willard L. Miranker
Yale University
Department of Computer Science
TR-1215

The Renormalization of Information

Willard L. Miranker[1]
Yale University Department of Computer Science
New Haven, CT

Abstract
Consciousness is a process involving both a measuring device and an observer. A neural net model that characterizes the neuron as the measuring device and the Hebbian synaptic dynamics as the observer is used to characterize a basic form of experience at the neuronal level. Methods of statistical mechanics are used to develop a token of this experience that is the neural analog of the spin correlation in the physics of magnetism. Renormalization group methodology is used to show how the phenomenon of experience at the level of a single neuron ramifies to characterize experience at the level of assemblies of neurons. Using that methodology we also develop the higher level token and infer that this ramification process is the route to explaining our consciousness. The possible role of the associated phase change phenomena in consciousness is discussed.
Key words: Renormalization group, consciousness, experience, measurement, neural nets, Hebbian dynamics, neuronal assemblies, change of phase, observer, spin

1 Introduction

Who or what is experiencing the feeling that characterizes our consciousness? This age old question expresses the intractability of the problem with which students of consciousness have always been confronted[2]. All of the proposed solutions, among them the homunculus or some other extra material reality have their well-known shortcomings[3] (Chalmers (1996), Churchland (1988), Penrose (1994), Searle (1994), Stapp (1996)).

---

[1] Miranker@cs.yale.edu

[2] Early writing on consciousness is found in Aristotle.

[3] Note that appeals to extra material reality are positioned in key places throughout science and are typically not explicit. The many examples include (i) the Shroedinger wave function which, having no existence in reality, can be neither measured nor observed, (ii) unmediated action at a distance exemplified by Newton's law of gravity and also by the Coulomb law, (iii) any of the counter factual effects of quantum mechanics, for example the non-locality of space/time (i.e., what Einstein termed 'spooky action at a distance') illustrated by both the double slit experiment and the interferometer experiment.

What we note here is that this experiencing is a form of a common process in both nature and culture, namely an act of making a measurement. A measurement has two aspects of interest for us. One is a device (the measuring instrument), and the second is an observer of the activity of the device. We shall characterize neuronal activity as a measurement process. The unconscious processing/transmission of signals by neuronal circuitry embodies both the quantity to be measured (the signals) and the measuring instrument (the neuron). It is the Hebbian dynamics that plays the role of the observer, as we shall see. These dynamics specify the adjustments to be made to the neuron's synaptic weights, those adjustments reflecting correlation between (i.e., observation of) the inputs and outputs of the measuring device. We shall take this Hebbian observer aspect of the neuronal measurement process as a basic form of consciousness. Naturally this basic level phenomenon of sensation bears only modest resemblance to what we experience personally as consciousness. The latter arises out of the former by sequential ramification of the measurement process produced by a finite hierarchy of assemblies of large numbers of intercommunicating neuronal circuits. (This process is entirely different from the homunculus style and its unbounded regress dilemma, as we shall see.)

The methodology we use is taken from statistical mechanics and the Renormalization group theory. The techniques of statistical mechanics have already been extensively applied to the study of neural nets (Amit, Gutfreund, Sampolinsky (1987), Hertz, Krogh, Palmer (1991), Hopfield (1982), Mezard, Parisi, Virasoro (1987)). These impressive applications dwell for the most part on the treatment of a static network (i.e., one with fixed synaptic weights). Here we employ this approach and augment it with a statistical mechanics-like consideration of the Hebbian dynamics for synaptic weight change that is central to our objective.

The Renormalization group technique (as originally applied to magnetic spin phenomena) uses the hierarchy of coarsening into blocks of spins of a ground model, more or less, existentially to produce so-called non-analyticities. The latter, occurring at the fixed-points of the Renormalization group transformation, characterize phase transitions of the ground model, as is well known (Goldenfeld (1992)). Here we give attention to the coarsening into circuits of neuronal assemblies to be the vehicle of ramification of the Hebbian dynamics from its synaptic/neuronal level to the neuronal assembly level. This ramification technique is the route from primitive levels of sensation arising at the synaptic level toward the forms of experience familiar to us as consciousness.

In Section 2 we introduce the neural net model and characterize its operation as a measurement process and as a primitive form of awareness at the synaptic/neuronal level.

In Section 3 we review the statistical mechanics methodology applied to the neural net in the presence of noise. In Section 4 we extend that application to characterize Hebbian dynamics as an observer process, and develop a token of consciousness (which turns out to be the neural net analog of the magnetic spin correlation function). In Section 5 we review the use of Renormalization group methodology for Ising block spins, and we reinterpret it as a way to ramify the primitive form of awareness to higher levels of neuronal organization. Speculations on the role of the associated phase changes in consciousness are made. In Section 6 we extend use of the Renormalization technique to a fully general hierarchy of neuronal assemblies. There results both an equation that specifies the fixed points which characterize phase changes as well as an expression for what we call feeling at the assembly level. Concluding we suggest why it is plausible that the token of our actual consciousness has the structure of this last expression.

2 Neuronal Dynamics as a Measurement Process

2.1 Network of McCulloch-Pitts Neurons

The information flow in a network of N McCulloch-Pitts neurons is described by the following input/output dynamics.

$$(2.1)\quad S_i^{(t+1)} = \mathrm{sgn}\left(\sum_{j=1}^{N} w_{ij} S_j(t) + h_i^{ext}\right), i = 1,\ldots,N,$$

where

$$(2.2)\quad \mathrm{sgn}(x) = \begin{cases} 1, x \geq 0 \\ -1, x < 0. \end{cases}$$

Here S_i is the output of neuron i. The synaptic connections from each of the N neurons to neuron i are characterized by the vector of synaptic weights $w_i = (w_{i1},\ldots,w_{iN})$. h_i^{ext} represents the exogenous input to neuron i. For convenience here and hereafter we omit the specification of index ranges when meaning is clear.

2.2 Hebbian dynamics

Information (e.g., a memory) is taken to be encoded (associatively) in the neural net by means of its synaptic weights. As the neurons conduct information processing according to (2.1), the synaptic weights change according to Hebbian dynamics. Namely,

$$(2.3) \quad \frac{dw_{ij}(t)}{dt} = \mathcal{H}(S_i(t), S_j(t-1)),$$

where $\mathcal{H}$ is the so-called Hebb function. Hebb's proposal may be expressed by saying that $\mathcal{H}$ is to have the sign of the correlation of its two arguments. A specification (among many other possibilities) of $\mathcal{H}$ which achieves this is made in Section 4.1.

2.3 The measurement/observation process

Using Figure 2.1, we characterize the pair of interrelated dynamical systems (2.1) - (2.3) as a measurement/observation process.

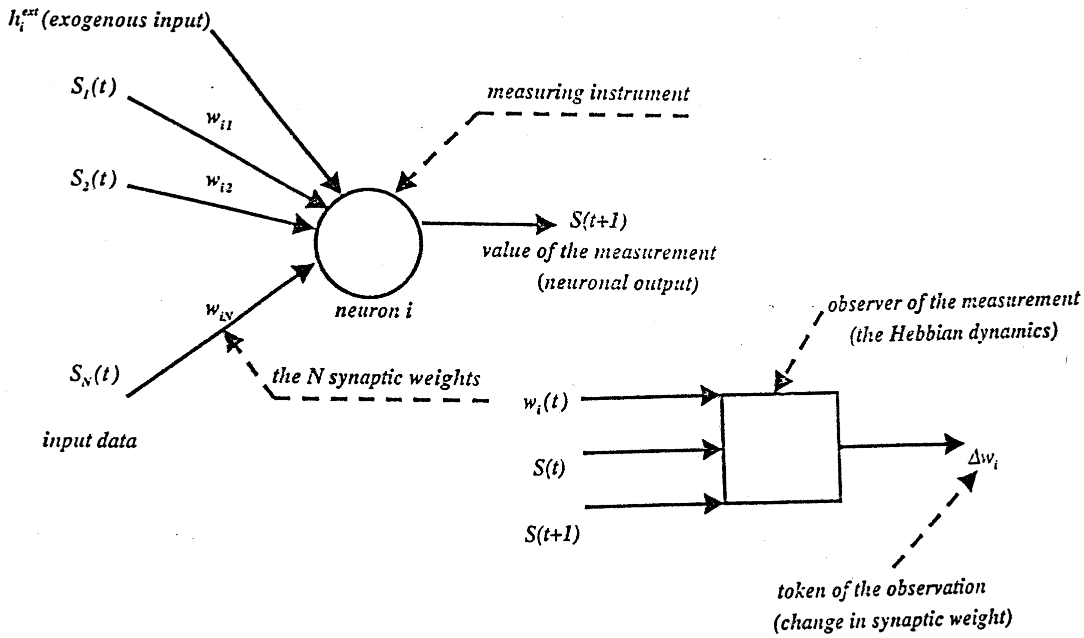

Fig. 2.1: The two neural net dynamical systems showing the token of observation

The input data is the vector $S(t) = (S_1(t),...,S_N(t))$. The measuring instrument is the neuron. The value of the measurement is the neuronal output $S_i(t)$ specified by (2.1). The observer of the measurement[4] is the process that generates the Hebbian dynamics specified by (2.3).

[4] It is interesting to note that the act of measurement in quantum mechanics (associated with the collapse of the wave function) is caused by the consciousness of a human observer when the latter notes the value of the measurement produced by some instrument (this according to Von Neumann (1995). See also Wigner (1961).) The picture here has some similarity to this point of view in the sense that the synaptic weights and the process characterized by the Hebbian dynamics are an aspect of the observer observing himself.

Discretizing the time, we describe the Hebbian observation process in terms of the incremental change Δw_{ij} in synaptic weight.

(2.4) $\quad \Delta w_{ij} = \mathcal{H}(S_i(n), S_j(n-1)).$

2.4 The token of awareness

We introduce the following terminology.

> Δw_{ij} given in (2.4) represents the degree to which the $j-th$ input synapse is *aware* of the measurement as a result of the observer process.

Although Δw_{ij} is only a *token of the awareness* which itself is the entire process represented by the Hebbian dynamics (2.4), we shall for convenience sometimes refer to it as the awareness itself[5].

3 Neuronal Dynamics (Measurement) in the Presence of Noise

In the presence of noise, the neuronal output (2.1) becomes stochastic. This is modeled by the following stochastic output rule[6].

(3.1) $\quad \Pr(S_i = \pm 1) = \dfrac{1}{1+\exp(\mp 2\beta h_i)}.$

Here $\beta = 1/T$, where T is a (pseudo-) temperature, a parameter controlling the noise level. Let h_i denote the total input to neuron i.

(3.2) $\quad h_i = \sum_j w_{ij} S_j + h_i^{ext}.$

---

[5] In a previous work (Miranker (2000)), a quantity related to this concept of awareness was called an atom of awareness, and it was viewed as a non-reducible property of matter. Here we avoid such a stand and allow that this awareness concept might very well be a reducible phenomenon. In the end, of course, the fundamental laws of physics in terms of which the Hebbian dynamics might be expressed are themselves non-reducible. Compare footnote 3.

[6] The illustrative material in this section is adapted from the discussion of the Hopfield model of a neural net in Herz, Krogh, Palmer (1991) chap. 2.

The exogenous input h_i^{ext} to neuron i may be viewed as coming from another neural network, perhaps that of a sense organ.

Now suppose that the neural net has been used to store a set of p so-called fundamental memories.

$$(3.3) \quad \xi^\mu = (\xi_1^\mu, \ldots, \xi_N^\mu), \mu = 1, \ldots, p.$$

This means that such a neural net when cued by an input close enough to $h_i^{ext} = \xi_i^\mu$ will produce $S_i = \xi_i^\mu$ as a steady state output. This may be accomplished by specifying the synaptic weights as follows (Haykin (1999) Chap. 2).

$$(3.4) \quad w_{ij} = \frac{1}{N}\sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu.$$

3.1 Mean field approximation

Let us take a so-called mean field approximation. That is, we replace h_i by its expected value $\langle h_i \rangle$, where in particular, using (3.2), we have

$$(3.5) \quad \langle h_i \rangle = \sum_j w_{ij} \langle S_i \rangle + h_i^{ext}.$$

Now using (3.1), we find that

$$(3.6) \quad \begin{aligned} \langle S_i \rangle &= (+1) Pr\{S_i = 1\} + (-1) Pr\{S_i = 1\} \\ &= tanh\, \beta \langle h_i \rangle. \end{aligned}$$

Combining (3.5) and (3.6) gives

$$(3.7) \quad \langle S_i \rangle = \tanh(\beta \sum_j w_{ij} \langle S_j \rangle + \beta h_i^{ext}).$$

Combining this in turn with (3.4) gives

$$(3.8) \quad \langle S_i \rangle = tanh(\frac{\beta}{N} \sum_{j,\mu} \xi_i^\mu \xi_j^\mu \langle S_j \rangle + \beta h_i^{ext}).$$

Let us suppose that the exogenous input is proportional to one of the stored patterns.

$$(3.9) \quad h_i^{ext} = h\xi_i^\nu,$$

where h is a constant. (One way to view this is to suppose that the exogenous signal, coming from a sense organ, closely resembles an earlier such recorded signal, in particular, the fundamental memory ξ^ν, say.) This motivates the hypothesis that $\langle S_i \rangle$ is itself proportional to the same stored pattern. In particular, we suppose that

$$(3.10) \quad \langle S_i \rangle = m\xi_i^\nu,$$

for some constant m. Inserting (3.9) and (3.10) into (3.8) gives

$$(3.11) \quad m\xi_i^\nu = \tanh\left(\frac{\beta}{N}\sum_{j,\mu} \xi_i^\mu \xi_j^\mu m\xi_j^\nu + \beta h\xi_i^\nu\right).$$

Next assume that N is large compared to p, so we may neglect the cross-talk term in (3.11), which then becomes

$$(3.12) \quad m\xi_i^\nu = \tanh\left[\beta(m+h)\xi_i^\nu\right].$$

Since $\xi_i^\nu = \pm 1$, we may write (3.12) as

$$(3.13) \quad m = \tanh \beta(m+h).$$

If $\xi_i^\nu = 1$, we say that the output $S_i = 1$ of neuron i is correct, and the output $S_i = -1$ is incorrect. If $\xi_i^\nu = -1$, we say that the output $S_i = -1$ is correct, and the output $S_i = 1$ is incorrect. Then by definition

$$(3.14) \quad \frac{\langle S_i \rangle}{\xi_i^\nu = \pm 1} = \frac{\Pr\{S_i = 1\} - \Pr\{S_i = -1\}}{\pm 1}.$$

Then combining (3.14) with (3.10), we conclude that

$$(3.15) \quad \frac{m+1}{2} = \Pr\{output\ \ S_i\ \ is\ \ correct\}.$$

Then the expected number of correct outputs of the neural net (the expected number of bits in the fundamental memory ξ^{ν} which are recalled correctly) in response to an arbitrary cue is

$$(3.16)\quad \langle N_{correct}\rangle = \frac{1}{2}N(1+m).$$

In Figure 3.1 we plot $m = m(h)$ versus T given by (3.13) for different values of h.

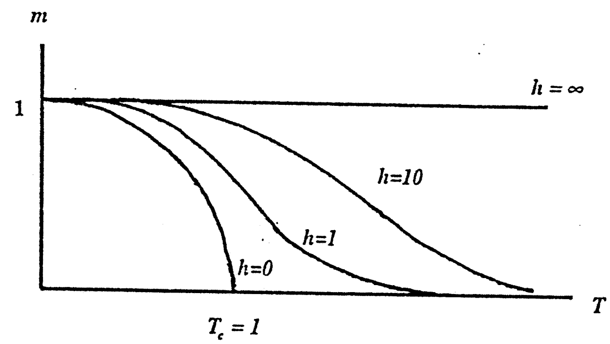

Fig. 3.1: A plot of *m(h,T)* demonstrating phase change in the retrieval process

For $h = 0$, we see a change of phase-like effect in Figure 3.1. Namely for T greater than a critical temperature, $T_c = 1$, we have $m = 0$, so that the expected number of correct bits in the retrieved memory is $N/2$, i.e., a random result. As T falls below T_c and approaches zero, there is a rapid rise of m to N, i.e., to full correctness of the expected number of correctly retrieved bits. For any temperature $T \geq 0$, the expected number of correctly retrieved bits increases with the strength h of the exogenous input and approaches N (full correctness) as $h \to \infty$.

4 Hebbian Dynamics (Observation Process) in the Presence of Noise

4.1 Expected value of observation

A simple form for $\mathcal{H}(x, y)$ that characterizes the correlation properties of synaptic weight development enunciated by Hebb, is the product form, namely

$$(4.1)\quad \mathcal{H}(x,y) = \kappa(x - \langle x\rangle)(y - \langle y\rangle),$$

where κ is a constant. We may take $\kappa = 1$ without loss of generality, by means of a change of time scale in (2.3). Now combining (4.1) with (2.4) and taking expected values, we find that

(4.2) $\langle \Delta w_{ij} \rangle = \langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle.$

So the expected value of awareness (literally, the token of observation) is proportional to the neural output correlation function. (In the case of modeling magnetic spins, the right member of (4.2) is referred to as the spin correlation function.)

The terminology introduced in Section 2.2 (concerning awareness of the neural processing by a synapse, that processing and that awareness interpreted as a measurement process) motivates the following definition.

Definition. We say that during a step of neural processing, the $ij-th$ synapse experiences a *sensation* σ_{ij}, where

(4.3) $\sigma_{ij} = \langle \Delta w_{ij} \rangle.$

We stress that while the members of (4.3) are tokens of awareness, awareness is properly a property characterizing the behavior of the $ij-th$ synapse (indeed the entire Hebbian dynamics process) as an observer of the neuronal measurement process.

4.2 Neural dynamics as a communication process, mirroring

Let

(6.22) $I_i = \frac{1}{N} \sum_j \sigma_{ij},$

and let

(6.23) $\bar{S} = \frac{1}{N} \sum_j S_j.$

In the case of modeling magnetic spins, $sig(\bar{S})$ is referred to as the block spin. Using (4.2) - (4.5), we find

(4.6)
$$\begin{aligned} I_i &= \frac{1}{N} \sum_j \left[\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle \right] \\ &= \langle \bar{S} S_i \rangle - \langle \bar{S} \rangle \langle S_i \rangle. \end{aligned}$$

We make the following stochastic independence hypothesis.

(4.7) $\langle \bar{S} S_i \rangle = \bar{S} \langle S_i \rangle.$

With this (4.6) becomes

(4.8) $I_i = (\bar{S} - \langle \bar{S} \rangle)\langle S_i \rangle.$

Combining this with (3.10), we find

(4.9) $$\begin{aligned} I_i &= (\bar{S} - \langle \bar{S} \rangle) m \xi_i^\nu \\ &= \pm(\bar{S} - \langle \bar{S} \rangle) m. \end{aligned}$$

Thus I_i is proportional to m, the expected value of neuronal output $\langle S_i \rangle$. We call this a *mirroring* of the expected value of the neuronal output by I_i.

4.3 Mutual information

Since many different patterns of input synaptic activity can correspond to each one of the two neuronal output values, $S_i = \pm 1$, all we can expect to tell about the neuronal input pattern from the value of the output is the average (counting signature) of the inputs. The mirroring expressed by (4.9) tells us this. So I_i is the *mutual information*[7] of neuronal input/output dynamics (subject to noise) interpreted as a step in a communication process (an information transmission process). Note that since we employ spin-valued variables, the mutual information here is a signed quantity. This can be avoided by replacing I_i by $(1 + I_i)/2$ (i.e., by going to binary valued variables). (Compare Miranker (2000)).

4.4 Neuronal feeling, the observer

Since I_i is an appropriate signed average of the sensations σ_{ij} of the input synapses of neuron i, it is a token of sensation of the entire neuron, which we shall refer to as the *feeling* of the neuron. (We stress once more the need to *differentiate between a quality and its token.*) Notice from (4.9) that neuronal feeling (which is the mutual information of the neuronal measurement process) may be attractive or repulsive.

---

[7] Recall that the mutual information is the uncertainty about an input that is resolved by knowledge of the output.

Indeed this feeling I_i is expressed in terms of the (externally/third person) measurable but unconscious neuronal output activity (cf. (4.9) f), that activity encoding the neural information being processed. So in this sense neuronal feeling is a non observable reflection of that information redounding directly from the so-called observer aspect of the measurement process which is neural dynamics (cf. Figure 2.1), in particular redounding directly from the Hebbian synaptic dynamics.

4.5 Influence of noise on feeling

Now let us recall from Figure 3.1, that $m = m(h)$. Indeed in the absence of an exogenous input (i.e., when $h = 0$), we have $m = 0$ for $T \geq T_c$, and so according to (4.9), there is no neuronal feeling in this case. On the other hand such feelings do return and grow stronger even at higher temperatures with the strengthening of the exogenous input. Should the input be removed, a generated feeling is preserved by the neuron only if the temperature is below critical. So if the noise level is below critical, the neural net can retain the feeling of a retrieved memory after the trigger for that retrieval has been removed.

4.6 An analogy between feeling and magnetization

Except for the inclusion of the Hebbian dynamics, the neuronal modeling discussed here is a well-known variant of the mean field analysis of the Ising model of magnetization. Indeed the quantity $\overline{S}$ in (4.6) is the magnetization M of the system in the corresponding Ising model. This analogy describes the feeling (defined here) associated with the neurons in a net as corresponding to the magnetization surrounding the dipoles composing a magnet. Thus our model suggests that *consciousness is a field of feeling associated with a neural net.* In what sense, if any, this field might surround the net (as the magnetic field surrounds the magnet), we can not yet say. Perhaps like the quantum mechanical probability amplitude, it is a field without existence in reality, a field that can be neither measured nor observed externally.

4.7 Feeling and free energy

The energy function of the neural net is defined as

$$(4.10) \quad H\{S\} = -\tfrac{1}{2}\sum_{i,j} w_{ij} S_i S_j - \sum_i h_i^{ext} S_i.$$

Defining the trace operator Tr_S as

(4.11) $Tr_S = \sum_{S_1=\pm1} \dots \sum_{S_N=\pm1}$,

we have for Z, the partition function of the net

(4.12) $Z = Tr_S \exp(\sum_{i,j} w_{ij} S_i S_j + \beta \sum_i h_i^{ext} S_i)$.

The free energy of the net is $F = -T \log Z$. We also have (Herz, Krogh, Palmer (1991) p.277)

(4.13) $\langle S_i S_j \rangle = -\dfrac{\partial F}{\partial w_{ij}}$,

and

(4.14) $\langle S_i \rangle = -\dfrac{\partial F}{\partial h_i^{ext}}$.

Combining (4.13) and (4.14) with (4.6), the feeling of neuron i is given in terms of the free energy as follows.

(4.15) $I_i = \dfrac{1}{N} \sum_j \left(\dfrac{\partial F}{\partial w_{ij}} - \dfrac{\partial F}{\partial h_i^{ext}} \dfrac{\partial F}{\partial h_j^{ext}} \right)$.

This displays the sensitivity of feeling to changes both in h^{ext} (let's say to exogenous stimulation) and in w_{ij} (let's say to episodic development).

5 Renormalization and the Ramification of Feeling

Consider an assembly of B neurons, the $i-th$ neuron in the assembly having feeling I_i, $i = 1,\dots,B$. Is there a feeling associated with the assembly, one which arises as a ramification of the B individual neuronal feelings? To illustrate how this comes about, we shall again adapt from the statistical mechanics analysis of the Ising model, this time employing the Renormalization Group technique. The latter is a coarse graining transformation endowing a block level magnetic structure to blocks of spins induced by the original individual spin interactions. Here we shall translate this analysis in the special case of magnetic spins with nearest neighbor interactions and uniform connection coefficients to our neuronal context. This (the customary case treated in the analysis of magnetic spins of the Ising model) allows us to exhibit the ramification of feelings that we seek. Developments of greater generality are given in Section 6. It is essential to note in the following developments that the characterization of feelings at increasingly higher levels of ramification in no way encounters the infinite observer regress dilemma of the homunculus concept. Our finite process has a totally different point of view since (among other reasons) its observer metaphysics are in place from the outset (Section 2.4).

Define a (virtual) output for a neuronal assembly by the majority rule[8].

$$(5.1) \qquad S = sig\sum_{i=1}^{B} S_i .$$

We suppose that the neurons are regularly spaced in a d-dimensional ($d = 1,2,3$) lattice with lattice spacing a. We suppose that the cell assembly is a block with block spacing taken to be a multiple la of the original lattice spacing, so that each assembly contains l^d neurons. The total number of assemblies is Nl^{-d}, where N is the total number of neurons.

The argument proceeds in terms of fixed points of the Renormalization group transformation. Then let T^* be the value of the temperature at such a fixed point, and let

$$(5.2) \qquad \tau = \frac{T - T^*}{T^*}.$$

(Recall that T is not a true temperature, but simply a parameter measuring the noise level.) Suppose that the corresponding normalized temperature associated with the assemblies has the form

$$(5.3) \qquad \tau_l = tl^{y_\tau},$$

where $y_\tau \geq 0$ is to be specified. Suppose further that the exogenous input for the assemblies has the form

$$(5.4) \qquad h_l = hl^{y_h},$$

where $y_h \geq 0$ is to be specified. Let r denote the physical location in space of a neuron. Then for the sensation (cf. (4.3)) we write

[8] The illustrative material in this section is adapted from the discussion on Ising model spins in Goldenfeld (1992) Chap. 9.

magnetic spins of the Ising model) allows us to exhibit the ramification of feeling that we seek. Developments of greater generality are given in Section 6. It is essential to take notice in the following developments that the characterization of feelings at increasingly higher levels of ramification in no way encounters the infinite observer regress dilemma of the homunculus. Our finite process has a totally different point of view. Moreover its metaphysics are already in place (cf. Section 2.4).

Define a (virtual) output for a neuronal assembly by the majority rule[8].

(5.1) $$S = sig\sum_{i=1}^{B} S_i.$$

We suppose that the neurons are regularly spaced in a d-dimensional ($d = 1,2,3$) lattice with lattice spacing a. We suppose that the cell assembly is a block with block spacing taken to be a multiple la of the original lattice spacing, so that each assembly contains l^d neurons. The total number of assemblies is Nl^{-d}, where N is the total number of neurons.

The argument proceeds in terms of fixed points of the Renormalization group transformation. Then let T^* be the value of the temperature at such a fixed point, and let

(5.2) $$\tau = \frac{T - T^*}{T^*}.$$

(Recall that T is not a true temperature, but simply a parameter measuring the noise level.) Suppose that the corresponding normalized temperature associated with the assemblies has the form

(5.3) $$\tau_l = tl^{y_\tau},$$

where $y_\tau \geq 0$ is to be specified. Suppose further that the exogenous input for the assemblies has the form

(5.4) $$h_l = hl^{y_h},$$

where $y_h \geq 0$ is to be specified. Let r denote the physical location in space of a neuron. Then for the sensation (cf. (4.3)) we write

[8] The illustrative material in this section is adapted from the discussion on Ising model spins in Goldenfeld (1992) Chap. 9.

(5.5) $\sigma_{ij} = G(|r_i - r_j|, \tau, h),$

which exhibits the dependence of σ_{ij} on its arguments. Then for the sensation of an assembly we have the following relation (Goldenfeld (1992) p. 270 ff.)).

(5.6) $G(\frac{|r_i - r_j|}{l}, \tau l^{y_\tau}, h^{y_l}) = \theta G(|r_i - r_j|, \tau, h)$

where

(5.7) $\theta = (l^d / l^{y_h})^2.$

In the case d=2, B=3 and a triangular lattice ($l = \sqrt{3}$), it is known that $y_t = 1$ and $y_h = \frac{15}{8}$. Then is this case we have $\theta = 3^{1/8} \approx 1.15$. This shows that the block or neuronal assembly sensation is stronger by a factor of 1.15 approx. than the sensation at the underlying neuronal level. Iterating the coarsening (ramification) process would successively increase the value of the sensation. Indeed every $8 / \log_2 3 \approx 5$ iterations would double the sensation magnitude.

5.1 Speculations on change of phase

The two dimensional Ising model with a triangular block coarsening and nearest neighbor interaction with uniform connection strength to which we have just referred, while rich enough to characterize an essential variety of magnetic phenomena, is an extreme specialization among possible arrangements of neural assemblies. The narrowness of this model is all the more exacerbated since we require Hebbian dynamics to be invoked to alter the synaptic weights (the latter being generalizations of the magnetic spin interactions.) While these defects are addressed in Section 6, let us first note that there are a number of possible features which a Renormalization group study of a more realistic neural net model might reveal which could impact the issues of ramification of synaptic/neuronal level feeling.

1. The fixed-point structure of the Renormalization group transformation in the more general (neural net) case is likely to be richer than in the special case corresponding to the Ising model treated here. This in turn will lead to a phase diagram for the neural net that is more complex than in the special Ising model case. It is those *occult phases* of information of the neural net that we speculate characterize the features of consciousness apparent to our experience.

2. The block structure (a virtual neural net of cell assemblies) is the fabric over which the features of ramified feelings is supported. According to the Renormalization group theory, these share the same critical point/phase space structure as the original neurons. Thus the so-called occult phases are available to the ramified feelings.

6 The Renormalization of Neuronal Assemblies

Now we conduct a Renormalization group transformation for a neural assembly, eliminating all of the restrictions of the model case treated in the previous Section 5. That is, we shall allow arbitrary connections among neurons, both with respect to synaptic (spatial) connectivity and synaptic weight. (This generalizes the restriction to nearest neighbor interaction with uniform connection strength of the Ising model.) The blocks become subassemblies of neurons within the original neural net, each block of arbitrary size and spatial configuration. We shall derive an equation for the fixed-points of the corresponding Renormalization transformation. We shall also derive expressions for the subassembly feeling (the generalization of the block spin correlation function). Hereafter we shall refer to the subassemblies of the original net simply as assemblies.

6.1 Derivation of the fixed-point equation

To begin consider the net of McCulloch-Pitts neurons introduced in Section 2 and its associated energy function (4.10). Divide the net into an arbitrary collection of neuronal assemblies. Let

(6.1) $$\sigma_I = \{S_1^I, S_2^I, \ldots\}$$

be the set of outputs of the neurons in assembly I. Let

(6.2) $$S_I = sig\{S_1^I + S_2^I + \ldots\}$$

be the virtual output of assembly I. (This is the analog of the block spin in the Ising model.) Let $H'\{S_I\}$ be the energy of the collection of assemblies. Then the collection partition function is

(6.3) $$Z = e^{H'\{S_I\}} = \sum_{\sigma_I} e^{H\{S_I, \sigma_I\}},$$

where we have written the net's energy function $H\{S_I\}$ as $H\{S_I, \sigma_I\}$. We proceed to estimate $H'\{S_I\}$ and begin with the case $h^{ext} = 0$. Write

(6.4) $H = H_0 + V,$

where H_0 corresponds to the intra-assembly energy and V corresponds to the inter-assembly energy. Then

$$(6.5)\quad \begin{aligned} H_0 &= \sum_I \sum_{i,j\in I} w_{ij} S_i S_j \\ &\equiv \sum_I S^I W^{II} S^I. \end{aligned}$$

and

$$(6.6)\quad \begin{aligned} V &= \sum_{I\neq J} \sum_{i\in I, j\in J} w_{ij} S_i S_j \\ &\equiv \sum_{I\neq J} S^I W^{IJ} S^J. \end{aligned}$$

Here $S^I = (S_1^I, S_2^I, \ldots)$ is a vector composed of the elements of the set σ_I. W^{IJ} denotes the matrix of those synaptic weights connecting neuron j in assembly J to a neuron i in assembly I. Note also that we have simplified the notation and written (here and hereafter) that $i \in I$ signifies that i runs over the neuronal indices in the assembly I.

Now for any function $A(S_I)$, let us define

$$(6.7)\quad \langle A(S_I)\rangle_0 \equiv \frac{\sum_{\{\sigma_I\}} e^{H_0\{S_I,\sigma_I\}} A(S_I,\sigma_I)}{\sum_{\{\sigma_I\}} e^{H_0\{S_I,\sigma_I\}}}.$$

With this notation, we may rewrite (6.3) as

$$(6.8)\quad e^{H'\{S_I\}} = \langle e^V \rangle_0 \sum_{\{\sigma_I\}} e^{H_0(S_I,\sigma_I)}.$$

Now let W_I denote the collection of synaptic weights within assembly I. Then using (6.5) we may write the partition function of assembly I as follows.

$$(6.9)\quad Z_0(W_I) = \exp \sum_{i,j\in I} w_{ij} S_i^I S_j^I.$$

If M is the total number of assemblies, then

(6.10) $\sum_{\{\sigma_I\}} e^{H_0\{S_I,\sigma_I\}} = \prod_{I=1}^{M} Z_0(W_I).$

Then from (6.8) and (6.10), we have

(6.11) $e^{H'\{S_I\}} = \langle e^V \rangle_0 \prod_I Z_0(W_I).$

Then

(6.12) $H'\{S_I\} = \sum_I \log Z(W_I) + \log\langle e^V \rangle_0.$

Let us now employ the cumulant expansion

(6.13) $\log\langle e^V \rangle_0 = \langle V \rangle_0 + \tfrac{1}{2}\left[\langle V^2 \rangle_0 - \langle V \rangle_0^2\right] + O(V^3).$

Then using (6.6), we have

(6.14) $\langle V \rangle_0 = \sum_{I \neq J} \langle S^I W^{IJ} S^J \rangle_0.$

Since H_0 does not itself couple different assemblies and is an average over outputs, the expression in (6.14) factorizes. Then

(6.15) $\langle V \rangle_0 = \sum_{I \neq J} \langle S^I \rangle_0 W^{IJ} \langle S^J \rangle_0.$

Using (6.5), (6.7) and (6.9), we have

(6.16) $\langle S^I \rangle_0 = \frac{1}{Z_0(W^{II})} \sum_{\{\sigma_I\}} e^{\sum_I S^I w^{II} S^I} S^I,$

where we have written $Z_0(W_I)$ as $Z_0(W^{II})$. Since $S_I S_I = 1, \forall I$, we may rewrite (6.15) as

(6.16) $\langle V \rangle_0 = \sum_{I \neq J} S_I [S_I \langle S^I \rangle_0 W^{IJ} \langle S^J \rangle_0 S_J] S_J.$

The fixed-point equation of the Renormalization group theory is obtained by equating w_{ij}, the coefficient of the interaction $S_i S_j$ in the energy of the original network (cf.

(4.10)) to the corresponding coefficient of the assembly level interaction $S_I S_J$ here. The latter is the bracketed expression in (6.17). Thus the equation we seek is

(6.18) $w_{ij} = S_I \langle S^I \rangle_0 W^{IJ} \langle S^J \rangle_0 S_J, \forall I, J, i \in I, j \in J, i \neq j.$

The averages $\langle S^I \rangle_0$ and $\langle S^J \rangle_0$ here are given by (6.16). Note that (6.18) is a system of simultaneous equations with one equation for every synaptic weight in the neural net.

6.2 Derivation of feeling at the assembly level

Employing the assembly level analogs of (4.13) and (6.3), we have

$$(6.19)\quad \begin{aligned} \langle S_I S_J \rangle &= \frac{\partial}{\partial W^{IJ}} \log Z \\ &= \frac{\partial}{\partial W^{IJ}} H'\{S_i\}. \end{aligned}$$

Then using (6.11), we obtain

$$(6.20)\quad \langle S_I S_J \rangle == \frac{\partial}{\partial W^{IJ}} \left[\sum_I \log Z_0(W_I) + \log \langle e^V \rangle_0 \right].$$

Since $Z_0(W_I) \equiv Z_0(W^{II})$, the first term in the bracket in (6.20) vanishes upon differentiation. Then employing the cumulant expansion (6.13) to its leading term and using (6.15), (6.20) becomes

(6.21) $\langle S_I S_J \rangle = (\langle S^I \rangle_0, \langle S^J \rangle_0).$

Then referring to (4.2) and (4.3), we find the following expression for the assembly level feeling.

(6.24) $\sigma_{IJ} = (\langle S^I \rangle_0, \langle S^J \rangle_0) - \langle S_I \rangle \langle S_J \rangle.$

Since the Renormalization process is formally repeatable, the token of our actual consciousness should have this same form.

REFERENCES

Amit, D., Gutfreund, H., Sompolinsky, H. (1987). Statistical Mechanics of Neural Networks near Saturation. Annals of Physics, **173**.

Chalmers, D. (1996). *The Conscious Mind: In Search of a Fundamental Theory*, New York, Oxford University Press.

Churchland, P. (1999). *Matter and Consciousness,* Cambridge, MIT Press.

Goldenfeld, N. (1992). *Lectures on Phase Transitions and the Renormalization Group*, Reading, Addison-Wesley.

Haykin, S. (1999). *Neural Networks, a Comprehensive Foundation.* Upper Saddle River, Prentice Hall

Hertz, J., Krogh, A., Palmer, R. (1991). *Introduction to the Theory of Neural Computation,* Redwood City, Addison-Wesley.

Hopfield, J. (1982). Neural Networks and Physical Systems with Emergent Collective Computational Abilities, PNAC, USA 79.

Mezard, M., Parisi, G., Virasoro, M. (1987). *Spin Glass Theory and Beyond,* Singapore, World Scientific.

Miranker,W. (2000). Consciousness is an Information State, J. Neural Parallel and Scientific Computation, **10**.

Penrose, R. (1994). *Shadows of the Mind,* New York, Oxford University Press.

Searle, J. (1994). *The Rediscovery of the Mind*, Cambridge, MIT Press.

Stapp, H. (1998). The Hard problem: a Quantum Approach, J. Consciousness Studies, 3.

Von Neuman, J. (1955). *Mathematical Foundations of Quantum Mechanics*, Princeton, Princeton University Press.

Wigner, E. (1961), Remarks on the Mind-Body Problem, in *The Scientist Speculates*, ed. I.J. Good, London, Heineman.