INTERPRETING LOGICS OF KNOWLEDGE IN PROPOSITIONAL DYNAMIC LOGIC WITH CONVERSE

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YALEU/DCS/TR 460
March, 1986

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
460

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
Interpreting Logics of Knowledge in Propositional Dynamic Logic with Converse

5. TYPE OF REPORT & PERIOD COVERED
Technical Report

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
Michael J. Fischer and Neil Immerman

8. CONTRACT OR GRANT NUMBER(s)
NSF: DCR-8405478
ONR: N00014-82-K-0154

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Department of Computer Science
Yale University
10 Hillhouse Avenue
New Haven, CT 06520

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS
NSF, 1800 G Street, Washington, DC 20550
ONR, 800 N Quincy, Arlington, VA 22217

12. REPORT DATE
March, 1986

13. NUMBER OF PAGES
12

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
Office of Naval Research
800 N Quincy
Arlington, VA 22217

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)
Approved for public release; distributed unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)
Logic of Knowledge
distributed system
modal logic
propositional dynamic logic

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)
A natural propositional logic of knowledge, common knowledge, and branching time appropriate for reasoning about distributed systems is presented. This logic may be interpreted in Propositional Dynamic Logic with Converse (PDLC), making the relationship between protocol models and general Kripke models precise and showing that PDLC already suffices for a certain amount of reasoning about knowledge in distributed systems. It follows that satisfiability for propositional logic of branching time remains EMPTIME complete with the addition of any combination of knowledge and common knowledge operators. Finally, the validity or satisfiability of a formula (over)

DD FORM 1473 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

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Interpreting Logics of Knowledge in Propositional Dynamic Logic with Converse

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March 5, 1986

Keywords: Logic of Knowledge, distributed system, modal logic, propositional dynamic logic.

1 Introduction

In this note we consider a natural propositional logic of knowledge, common knowledge, and branching time which is appropriate for distributed systems. We show that this language may be interpreted in Propositional Dynamic Logic with Converse (PDLC) [St81,Pr81]. This result makes the relationship between our protocol model and general Kripke models precise (cf. [FI85]) as well as showing that PDLC already suffices for a certain amount of reasoning about knowledge in distributed systems. It was already known that the satisfiability problem for propositional logic of branching time is EXPTIME complete, cf. [EH85]. As a corollary of our result we show that satisfiability for propositional logic of branching time remains EXPTIME complete with the addition of any combination of knowledge and common knowledge operators. (This last result has been independently obtained in [HV86].)

*Research supported in part by the National Science Foundation under Grant Number DCR-8405478 and by the Office of Naval Research under Contract Number N00014-82-K-0154.

†Research supported by an NSF postdoctoral fellowship and by the Mathematical Sciences Research Institute, Berkeley, CA.

2 Definitions

We define Propositional Temporal Knowledge Logic (PTKL) as follows. Let PROP $= \{S_1, S_2, \ldots\}$ be a set of propositional symbols. Let PART $= \{1, 2, \ldots, n\}$, $n \geq 2$ be a finite set of participants. Let $\Phi = \Phi(\text{PART}, \text{PROP})$, the formulas of PTKL, be the smallest set of strings containing PROP and closed under the following rules:

1. If $\alpha, \beta \in \Phi$ then so are $\neg\alpha$ and $\alpha \wedge \beta$.
2. If $\alpha, \beta \in \Phi$ then so are $Y\alpha$, $G\alpha$ and $(\alpha U \beta)$.
3. If $\alpha \in \Phi$ and $H \subseteq \text{PART}$ then $C_H\alpha \in \Phi$.

The intuitive meaning of the temporal operators is as follows: $Y\alpha$ means that α holds at every next step. $G\alpha$ means that α holds at all points in the future. $(\alpha U \beta)$ means that α is true and remains true *until* β becomes true.

We adopt abbreviations for the dual operators: $X\alpha \equiv \neg Y \neg \alpha$ meaning that α holds at some next step, and $F\alpha \equiv \neg G \neg \alpha$ meaning that α holds at some future step.

For H a singleton, $H = \{i\}$, we adopt the abbreviation $K_i\alpha$, read "i knows α," for $C_H\alpha$. The intuitive meaning is that α is true in all conceivable situations that are consistent with i's local view. In the more general case $C_H\alpha$ is read, "It is common knowledge among the members of H that α." This is precisely defined below. See also Fact 2.1 for an equivalent formulation.

The semantics of PTKL are defined using a kind of Kripke model called a distributed protocol. See [FI85] for a detailed discussion of this model. Let PROP be fixed. Define a ***protocol*** to be a tuple $\mathcal{P} = \langle n, Q, I, \tau, \pi \rangle$. PART$= \{1, \ldots, n\}$ is a set of ***participants***, Q is a set of ***local states***, and Q^n is the set of n-tuples called ***global states***. $I \subseteq Q^n$ is a set of ***initial global states***, the function $\pi : Q^n \times \text{PROP} \rightarrow \{0, 1\}$ evaluates the propositional letters at each global state, and $\tau \subseteq Q^n \times Q^n$ is the next move relation on global states. Let τ^* be the reflexive transitive closure of τ and define the ***reachable global states in $\mathcal{P}$*** to be

$$R_{\mathcal{P}} = \{q \in Q^n \mid \text{for some } s \in I, \langle s, q \rangle \in \tau^*\}.$$

Intuitively, a global state q is reachable if there is a τ-path $s, p_1, \ldots, p_{r-1}, q$ starting in an initial global state s and ending in q.

Given a protocol $\mathcal{P} = \langle n, Q, I, \tau, \pi \rangle$, a global state $q \in R_{\mathcal{P}}$, and a PTKL formula $\alpha \in \Phi$, we define the satisfaction relation $\langle \mathcal{P}, q \rangle \models \alpha$ in the usual way by induction on the complexity of α:

1. For $S \in$ PROP, $\langle \mathcal{P}, q \rangle \models S \Leftrightarrow \pi(q, S) = 1$.
2. $\langle \mathcal{P}, q \rangle \models Y\beta \Leftrightarrow$ (for all p)(if $\langle q, p \rangle \in \tau$ then $\langle \mathcal{P}, p \rangle \models \beta$).
3. $\langle \mathcal{P}, q \rangle \models G\beta \Leftrightarrow$ (for all p)(if $\langle q, p \rangle \in \tau^*$ then $\langle \mathcal{P}, p \rangle \models \beta$).
4. $\langle \mathcal{P}, q \rangle \models \beta U \gamma \Leftrightarrow$ (for all $n \geq 0$)(for all $p_0, p_1, \ldots, p_n$)(if ($q = p_0$ and for $i = 1, \ldots, n$, $\langle p_{i-1}, p_i \rangle \in \tau$ and $\langle \mathcal{P}, p_i \rangle \models \neg\gamma$) then $\langle \mathcal{P}, p_n \rangle \models \beta$).

The only unusual case occurs when $\alpha = C_H\beta$. For $i \leq n$, let $(q)_i$ denote the i^{th} component of q. Define the equivalence relation $\overset{i}{\sim}$ on $R_{\mathcal{P}}$ by

$$p \overset{i}{\sim} q \Leftrightarrow (p)_i = (q)_i \,.$$

For $H = \{i_1, \ldots, i_r\}$, let the equivalence relation $\overset{H}{\sim}$ be the transitive closure of $\left(\overset{i_1}{\sim} \cup \overset{i_2}{\sim} \cup \ldots \cup \overset{i_r}{\sim}\right)$. Finally we define:

5. $\langle \mathcal{P}, q \rangle \models C_H\beta \Leftrightarrow$ (for all p)(if $p \overset{H}{\sim} q$ then $\langle \mathcal{P}, p \rangle \models \beta$) .

From this definition it is straightforward to prove:

Fact 2.1 *[FI85] The following two statements are equivalent for any set $G \subseteq$ PART:*

1. $\langle \mathcal{P}, p \rangle \models \mathsf{C}_G\alpha$.

2. $(\forall r \geq 0)(\forall i_1, \ldots, i_r \in G)(\langle \mathcal{P}, p \rangle \models \mathsf{K}_{i_1}\mathsf{K}_{i_2} \ldots \mathsf{K}_{i_r}\alpha)$.

3 Main Results

In Theorem 3.1 below, we give an interpretation of PTKL in Propositional Dynamic Logic with Converse (PDLC) [St81]. It then follows using Pratt's EXPTIME decision procedure for PDLC [Pr81] that the satisfiability problem for PTKL is solvable in EXPTIME. This is Corollary 3.7. We then observe in Theorem 3.8 that if a PTKL formula is satisfied by some protocol in which at least two participants are mentioned, then it is satisfied by a protocol in which the only participants are those explicitly mentioned in the formula. Thus, allowing extra participants with "hidden" state does not increase the power of the system.

We assume that the reader is familiar with Propositional Dynamic Logic (PDL), see e.g. [FL79]. PDLC is PDL plus the convese operator: for each program a we let a^- denote its converse.

Theorem 3.1 *There is a simultaneously logspace and time $O(n^2)$ computable mapping f from formulas of PTKL to formulas in PDLC such that for all $\alpha \in \Phi$, α is satisfiable if and only if $f(\alpha)$ is satisfiable.*

The proof is contained in three lemmas. First we define the mapping f and show that it is easily computable. Next we show that if α is satisfiable, then so is $f(\alpha)$, and finally we show the converse, that if $f(\alpha)$ is satisfiable, then so is α.

Let PART $= \{1, \ldots, n\}$. The atomic program symbols we will need are $\{t, e_1, \ldots, e_n\}$. Symbol t will correspond to a τ step and the e_i's together with their converses will correspond to $\overset{i}{\sim}$ links. The function f is defined inductively as follows:

1. For $S \in$ PROP, $f(S) = S$.
2. $f(\neg\alpha) = \neg f(\alpha)$; $f(\alpha \wedge \beta) = f(\alpha) \wedge f(\beta)$.
3. $f(Y\alpha) = [t]f(\alpha)$; $f(G\alpha) = [t^*]f(\alpha)$;
   $f(\alpha U \beta) = [(t; \neg f(\beta)?)^*]f(\alpha)$.
4. For $H = \{i_1, \ldots, i_r\}$, $f(C_H\alpha) = [(e_{i_1} \cup e_{i_1}^- \cup \ldots \cup e_{i_r} \cup e_{i_r}^-)^*]f(\alpha)$.

Lemma 3.2 *f is simultaneously logspace and time $O(n^2)$ computable.*

Proof Straightforward using standard techniques. ∎

Lemma 3.3 *Given a protocol $\mathcal{P} = \langle n, Q, I, \tau, \pi \rangle$, there is a PDL structure $h(\mathcal{P})$, whose worlds are the reachable global states of $\mathcal{P}$, such that for any PTKL formula α and reachable global state p, $\langle \mathcal{P}, p \rangle \models \alpha$ iff $\langle h(\mathcal{P}), p \rangle \models f(\alpha)$.*

Proof We define the PDL structure $h(\mathcal{P})$ as follows: the set of worlds W of $h(\mathcal{P})$ is $R_\mathcal{P}$, and the mapping $\pi' : \mathrm{PROP} \to 2^W$ is given by $\pi'(S) = \{p \in R_\mathcal{P} \mid \pi(p, S) = 1\}$. For each participant i, the meaning of e_i is given by

$$\rho(e_i) = \{\langle p, q \rangle \in R_\mathcal{P} \times R_\mathcal{P} \mid p \overset{i}{\sim} q\},$$

and finally

$$\rho(t) = \{\langle p,q\rangle \in R_{\mathcal{P}} \times R_{\mathcal{P}} \mid \langle p,q\rangle \in \tau\}.$$

It is easy to show by induction on the complexity of α that for $p \in R_{\mathcal{P}}$,

$$\langle \mathcal{P},p\rangle \models \alpha \Leftrightarrow \langle h(\mathcal{P}),p\rangle \models f(\alpha).$$

We leave the details to the reader. ∎

Lemma 3.4 *Given a PDL structure $K = \langle W,\rho,\pi\rangle$ with atomic program symbols t, $e_1,\ldots,e_s$, there is a protocol $g(K)$ with $n = \max(s,2)$ participants and a surjection η from global states of $g(K)$ to worlds of K such that for any PTKL formula α and global state q, $\langle g(K),q\rangle \models \alpha$ iff $\langle K,\eta(q)\rangle \models f(\alpha)$.*

Proof For $1 \le i \le n$, let $\equiv_i$ be the reflexive, symmetric, and transitive closure of $\rho(e_i)$ on W if $i \le s$, and let $\equiv_i$ be the equality relation if $i > s$. Let $[w]_i$ denote the $\equiv_i$ equivalence class which contains w. Let $M = |W|$, and let $\omega : \{0,\ldots,M-1\} \rightarrow W$ be a bijection.

We define the protocol $g(K) = \langle n,Q,I,\tau,\pi'\rangle$ as follows. Let

$$Q = \{([w]_i,m) \mid w \in W,\ 1 \le i \le n,\ 0 \le m < M\}.$$

Define the map $\eta : Q^n \rightarrow W$ by

$$\eta(\langle([w]_1,m_1),([w]_2,m_2),\ldots,([w]_n,m_n)\rangle) = \omega(\textstyle\sum m_i \bmod M),$$

and let

$$\begin{aligned} I = \{&q \in Q^n \mid \text{for some } w \in W,\ m_1,\ldots,m_n \in \{0,\ldots,M-1\},\\ &q = \langle([w]_1,m_1),([w]_2,m_2),\ldots,([w]_n,m_n)\rangle \text{ and } \eta(q) = w\}.\end{aligned}$$

The idea here is that in I, each local state $([w]_i,m_i)$ has as first component the $\equiv_i$ equivalence class we are in and the second component gives no further information except when added to all the other m_j's, in which case it tells us exactly which world we are in and thus what the allowable next moves are.

To complete the definition of $g(K)$ let

$$\tau = \{\langle q,q'\rangle \in I \times I \mid \langle\eta(q),\eta(q')\rangle \in \rho(t)\},$$

and let

$$\pi'(q,S) = \begin{cases} 1 & \text{if } \eta(q) \in \pi(S) \\ 0 & \text{otherwise.}\end{cases}$$

Note that by definition, $\tau \subseteq I \times I$ and thus $R_{g(K)} = I$.

For any $H \subseteq$ PART, let $\equiv_H = (\bigcup_{i\in H} \equiv_i)^*$.

Fact 3.5 *Let $p, p' \in I$ with $p \overset{H}{\sim} p'$. Then $\eta(p) \equiv_H \eta(p')$.*

Proof First assume $H = \{i\}$. Let $p, p' \in I$ and let $w = \eta(p)$ and $w' = \eta(p')$. If $p \overset{i}{\sim} p'$, then the i^{th} components of p and p' are the same, so $[w]_i = [w']_i$. Hence, $\eta(p) = w \equiv_i w' = \eta(p')$. The extension to arbitrary H follows easily by induction on the minimal r such that $\langle p, p' \rangle \in \left(\bigcup_{i \in H} \overset{i}{\sim}\right)^r$. ∎

Fact 3.6 *Let $p \in I$, let $\eta(p) = w$ and let $w \equiv_H w'$. Then there exists $p' \in I$ such that $\eta(p') = w'$ and $p \overset{H}{\sim} p'$.*

Proof First assume $H = \{i\}$, and let p, w, and w' be as above. We may write $p = \langle([w]_1, m_1), \ldots, ([w]_n, m_n)\rangle$. Choose $k \neq i$, possible since $n \geq 2$. Let $p' = \langle([w']_1, m'_1), \ldots, ([w']_n, m'_n)\rangle$, where $m'_j = m_j$ for all $j \neq k$, and choose m'_k such that $\eta(p') = w'$. Thus, $p' \in I$, and since $[w']_i = [w]_i$ and $m'_i = m_i$, we have $p \overset{i}{\sim} p'$ as desired. The extension to arbitrary H follows easily by induction on the minimal r such that $\langle w, w' \rangle \in (\bigcup_{i \in H} \equiv_i)^r$. ∎

Returning to the proof of Lemma 3.4, we show by induction on the complexity of $\alpha \in \Phi$ that for $q \in R_{g(K)}$,

$$\langle g(K), q \rangle \models \alpha \quad \Leftrightarrow \quad \langle K, \eta(q) \rangle \models f(\alpha).$$

The only interesting case is when $\alpha = C_H \beta$. Let $H = \{i_1, \ldots, i_r\}$ and $q \in R_{g(K)}$. Then

$$\begin{aligned}
&\langle g(K), q \rangle \models C_H \beta \\
&\quad \Leftrightarrow \quad \text{for all } p \in R_{g(K)},\ \text{if } q \overset{H}{\sim} p \text{ then } \langle g(K), p \rangle \models \beta \\
&\quad \quad \text{(by definition of } C_H\text{)} \\
&\quad \Leftrightarrow \quad \text{for all } p \in R_{g(K)},\ \text{if } \eta(q) \equiv_H \eta(p) \text{ then } \langle K, \eta(p) \rangle \models f(\beta) \\
&\quad \quad \text{(by Facts 3.5 and 3.6 and the induction hypothesis)} \\
&\quad \Leftrightarrow \quad \text{for all } w' \in W,\ \text{if } \eta(q) \equiv_H w' \text{ then } \langle K, w' \rangle \models f(\beta) \\
&\quad \quad \text{(since } \eta \text{ is surjective)} \\
&\quad \Leftrightarrow \quad \langle K, \eta(q) \rangle \models [(e_{i_1} \cup e_{i_1}^- \cup \ldots \cup e_{i_r}^-)^*] f(\beta) \\
&\quad \quad \text{(by definition of } \equiv_i \text{ and PDLC).}
\end{aligned}$$

This completes the proof of Lemma 3.4 and of Theorem 3.1. ∎

Corollary 3.7 *The satisfiability problem for PTKL is decidable in EXPTIME.*

Given a PTKL formula α, let $H(\alpha)$ be the set of participants that appear in α. More precisely, if $C_{H_1}, \ldots, C_{H_r}$ are the knowledge operators that appear in α, then $H(\alpha) = H_1 \cup \ldots \cup H_r$. The following theorem shows that if there are at least two participants mentioned in a formula then adding extra participants not mentioned in the formula cannot affect its satisfiability. Note that this is nontrivial because the truth of a knowledge formula in a particular structure *can* be affected by participants not mentioned in the formula.

Theorem 3.8 *Let α be a satisfiable formula of PTKL. Then α is satisfiable in a protocol $\mathcal{P} = \langle n, Q, I, \tau, \pi \rangle$ in which $n = \max(|H(\alpha)|, 2)$.*

Proof Let α be satisfiable in a protocol $\mathcal{P}$, and let $n = \max(|H(\alpha)|, 2)$. We will show that α is satisfiable in a protocol with n participants. By Lemma 3.3, $f(\alpha)$ is satisfiable in the PDL structure $h(\mathcal{P})$. But $f(\alpha)$ only contains program letters t and e_i for $i \in H(\alpha)$. Hence, $f(\alpha)$ is also satisfiable in a PDL structure K containing only the relations $\rho(t)$ and $\rho(e_i)$ for $i \in H(\alpha)$. By Lemma 3.4, α is satisfiable in the protocol $g(K)$, which has only n participants. ∎

4 Hardness

The following theorem is very similar to the corresponding lower bound in [FL79]. Emerson and Halpern [EH85] already point out that this theorem can be proved in this way. We include the details for the sake of completeness.

Theorem 4.1 *Let M be an ASPACE(n) Turing machine. Then there is a logspace and $n \log n$ time computable function $d : \{0,1\}^* \to \Phi$ such that M accepts x iff $d(x)$ is satisfiable. Furthermore the operator C does not occur in $d(x)$.*

Proof

An instantaneous description (ID) of M for an input of length n will consists of $n+3$ symbols as follows: a left end-marker $\triangleleft$, n tape cells, a state symbol $q \in Q_M$ located immediately to the left of the cell being examined by M's head, and a right end-marker $\triangleright$. Let $\forall_M \subset Q_M$ be the set of M's universal states and let A_M be M's tape alphabet. Let $\Sigma = Q_M \cup A_M \cup \{\triangleleft, \triangleright\}$ be the alphabet of all possible symbols in an ID of M. We will assume without loss of generality that M has a clock which causes each computation branch to enter the unique rejecting state, q^{reject}, after c^n steps. We will also assume that there is a unique accepting state, q^{accept}.

Given an input $x \in \{0,1\}^n$, we let PROP $= \{\sigma_i \mid \sigma \in \Sigma$ and $-1 \leq i \leq n+1\}$. We will let $d(x)$ be the conjunction of the following PTKL formulas. Intuitively $d(x)$ will assert that each reachable global state determines an ID of M, that in particular the current global state determines M's initial ID on input x, that every global state leads in a next time step to at least one global state whose ID is a valid next move of M, that every global state corresponding to a universal ID leads in next time steps to each of the two possible next moves of M, and that the reject state never occurs. It thus follows that $d(x)$ is satisfiable if and only if M accepts x.

- $G(\bigwedge_{i=-1}^{n+1} \bigvee_{\sigma\in\Sigma}(\sigma_i \wedge \bigwedge_{\tau\neq\sigma} \neg\tau_i)) \wedge G(\triangleleft_{-1} \wedge \triangleright_{n+1})$, i.e. each cell i always contains exactly one symbol of Σ, and the end-markers are fixed.

- $q_0^{start} \wedge (\bigwedge_{i:x_i=0} 0_i) \wedge (\bigwedge_{i:x_i=1} 1_i)$, i.e. the initial ID is q^{start} followed by x.

- $G(\bigwedge_{\alpha,\beta,\gamma\notin Q_M} \bigwedge_{i=0}^{n}(\alpha_{i-1} \wedge \beta_i \wedge \gamma_{i+1} \rightarrow Y\beta_i))$, i.e. a cell not bordered by a state symbol is always preserved.

- $G(\bigwedge_{\beta\in Q_M} \bigwedge_{i=0}^{n}(\alpha_{i-1}\wedge\beta_i\wedge\gamma_{i+1} \rightarrow X(\alpha'_{i-1}\wedge\beta'_i\wedge\gamma'_{i+1} \vee \alpha''_{i-1}\wedge\beta''_i\wedge\gamma''_{i+1}))$, i.e. there is a next step that reflects at least one of the possible next moves of M.

- $G(\bigwedge_{\beta\in\forall_M} \bigwedge_{i=0}^{n}(\alpha_{i-1} \wedge \beta_i \wedge \gamma_{i+1} \rightarrow X(\alpha'_{i-1} \wedge \beta'_i \wedge \gamma'_{i+1}) \wedge X(\alpha''_{i-1} \wedge \beta''_i \wedge \gamma''_{i+1}))$, i.e. when we're in a universal state there are next steps reflecting each of the two possible next moves.

- $G(\bigwedge_{i=0}^{n} \neg q_i^{reject})$, i.e. we never enter the rejecting state.

It is not hard to verify that $d(x)$ meets the required conditions. ∎

Corollary 4.2 *The satisfiability problem for PTKL is EXPTIME complete even with only one participant and no occurrences of C_H.*

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